Experimental Physics II – Assignment No. 10

10.1: $\mu = 3000$

a) Since $\Phi_m = \int \vec{B}\, d\vec{A} = \vec{B}\cdot\vec{A} = BA$ ($\vec{B}$ is constant in this coil and perpendicular to the coil's cross-section)

and $\Phi_m = LI = \mu\mu_0 \frac{N^2}{l}\cdot A\cdot I$, we find:

$$B\cdot A = \mu\mu_0\frac{N^2}{l}I\cdot A \Rightarrow B = \mu\mu_0 \frac{N^2}{l} I$$, we l is the length of the coil.

$$\Rightarrow B(t) = \mu\mu_0\frac{N^2}{l}I(t) = 4\pi\cdot 10^{-7}\cdot\frac{100^2}{0{,}1}\cdot 0{,}2\, \cancel{A}\cdot\sin(2\pi\cdot 25\,Hz\cdot t)\cdot 3000$$

$$\approx 0{,}025\,T\cdot\sin(50\pi\, Hz\cdot t)\cdot 3000$$

b) By Maxwell's laws, $\oint_S \vec{E}\, d\vec{s} = -\int_A \frac{\partial\vec{B}}{\partial t}\, d\vec{A}$ must hold for any closed path S that encloses the area A. We choose S as the circle with radius r centered around the z-axis, and consider that all fields must exhibit rotational symmetry with respect to the z-axis because the generating coil exhibits said symmetry.

Thus, $\oint_S \vec{E}\, d\vec{s} = E\oint_S ds = -\frac{\partial B}{\partial t}\oint_A dA$ and therefore:

$$E\cdot 2\pi r = -\frac{\partial B}{\partial t}\pi r^2 \Rightarrow E(r,t) = -\frac{1}{2}\frac{\partial B}{\partial t}(r,t)\cdot r = -\frac{1}{2}\mu\mu_0\frac{N^2}{l}\frac{\partial I}{\partial t}(t)\, r$$

$$\Rightarrow E(r,t) = -\frac{1}{2}\mu\mu_0\frac{N^2}{l}I_0\, 2\pi f\cos(2\pi f t)\cdot r = -\mu\mu_0\frac{N^2}{l}I_0\pi f\cos(2\pi f t)\cdot r$$

$$\Rightarrow |E|(r,t) \approx 0{,}025\,T\cdot\pi\cdot 25\,Hz\cdot\cos(50\pi\cdot t)\cdot r \approx 1{,}874\,\frac{V}{m^2}\cos(50\pi\cdot t)\cdot r$$

$$= \mu\mu_0\frac{N^2}{l}I_0\pi f\cos(2\pi f t)\, r$$

c) The current density is given by $j = \sigma E$, where $\sigma = \frac{1}{\rho}$ with ρ as specific resistance.

$$\Rightarrow j(r) = \frac{1}{\rho}E(r) = \frac{1}{10^{-7}\,\Omega m}\cdot 1{,}874\,\frac{V}{m^2}\cos(50\pi t)\cdot r \approx 2{,}0\cdot 10^{7}\,\frac{A}{m^3}\cdot\cos(50\pi t)\cdot r\cdot 3000$$

d) ~~Since $P = U\cdot I = U\cdot\int j\, dA = \int E\, ds\cdot\int j\, dA$~~

Since $P = U\cdot I$, the infinitesimal change dP is ~~$dP = d(U\cdot I) = d($~~

$$dP = dU\cdot dI = E\cdot dl\cdot j\cdot dA = E\cdot j\cdot dl\cdot dA = E\cdot j\cdot dV$$

$\Rightarrow P(t) = \int_V E\cdot j\cdot dV$, where V is the volume enclosed by the coil.

$\Rightarrow P(t) = \int_{-l/2}^{l/2}\int_0^{r}\int_0^{2\pi} E\cdot j\cdot r'\, dr'\, d\varphi\, dz$ as $r'\, dr'\, d\varphi\, dz$ is the cylindrical volume element

$$= \int_{-l/2}^{l/2}\int_0^{r}\int_0^{2\pi}\frac{1}{\rho}E^2 r'\, dr'\, d\varphi\, dz = \frac{1}{\rho}\int_{-l/2}^{l/2}\int_0^{r}\int_0^{2\pi} E^2 r'\, dr'\, d\varphi\, dz = \frac{1}{\rho}\int_{-l/2}^{l/2}\int_0^{r} 2\pi E^2 r'\, dr'\, dz$$

$$= \frac{2\pi}{\rho}\int_0^{r} l E^2 r'\, dr' = \frac{2\pi l}{\rho}\int_0^{r} E^2(r')\, r'\, dr' = \frac{2\pi l}{\rho}\int_0^{r}\left(\mu\mu_0\frac{N^2}{l}I_0\pi f\cos(2\pi f t)\, r'\right)^2 r'\, dr'$$

$$= \frac{2\pi l}{\rho}\mu_0^2\frac{N^4}{l^2}I_0^2\pi^2 f^2\cos^2(2\pi f t)\int_0^{r} r'^3\, dr'\, \mu^2 = \frac{2\pi l}{\rho}\mu_0^2\frac{N^4}{l^2}I_0^2\pi^2 f^2\cos^2(2\pi f t)\,\frac{1}{4}r^4\mu^2$$

$$\Rightarrow P(t) = \underbrace{\frac{2\pi l}{2\rho}\mu_0^2\frac{N^4}{l^2}I_0^2\pi^2 f^2 r^4}_{=:P_0}\cos^2(2\pi f t)\cdot\mu^2$$

e) ~~$\langle P(t) \rangle$~~

The time average of a periodic function is the same as the time average over one period:

$$\langle P(t) \rangle = \frac{1}{T}\int_0^T P(t)\,dt = \frac{1}{1/f}\int_0^{1/f} P(t)\,dt = f\int_0^{1/f} P(t)\,dt = fP_0\int_0^{1/f}\cos^2(2\pi f t)\,dt$$

$$= fP_0\,\frac{1}{8\pi f}\left(4\pi f t + \sin(4\pi f t)\right)\Big|_0^{1/f}$$

$$= fP_0\,\frac{1}{8\pi f}\left(4\pi + \sin(4\pi) - 0 + \sin(0)\right)$$

$$= \frac{P_0}{2} = \frac{\ell}{8\rho}\mu_0^2\mu^2\frac{N^4}{\ell^2}I_0^2\pi^2 f^2 r^4 \approx 6{,}08\cdot 10^{-4}\,W \cdot 3000 \; \cancel{\approx 60{,}8\,mW}$$

$$\approx \underline{0{,}608\,mW}\cdot 3000 = \underline{1{,}826\,W}$$

10.2: (a) $\int \bar{H} d\bar{s} = I_{free} = NI$

$$\Rightarrow \oint_{Torus} \bar{H} d\bar{s} = H_0 L = NI \Rightarrow H_0 = \frac{NI}{L}$$

because $L \gg D$,
H is approx. homogenous.

$$B_0 = \mu_0 H_0 . \Rightarrow B_0 = 0{,}2\,T \Rightarrow 0{,}2T \cdot L \cdot \frac{1}{\mu_0 N} = I$$

$$= 0{,}2T \cdot 100\,cm \cdot \frac{1}{\mu_0 \cdot 500}$$

$$= \underline{318\,A}$$

(b) $\phi = \int \bar{B} d\bar{A}$.

$$\Phi_{Air} = \bar{B}_{Air} \int_{Air} d\bar{A} = B_{Air} \cdot \pi \left(D^2 - \left(\frac{D}{2}\right)^2\right) = \frac{3}{4} B_{Air} \pi D^2$$

$$\Phi_{Iron} = \bar{B}_{Iron} \int_{Iron} d\bar{A} = B_{Iron} \int_{Iron} dA = B_{Iron} \pi \cdot \left(\frac{D}{2}\right)^2 = \frac{1}{4} B_{Iron} \pi D^2 = \frac{1}{4} \mu B_0 \pi D^2$$

$$\Rightarrow \frac{\Phi_{Air}}{\Phi_{Iron}} = \frac{3}{\mu}$$

$B_0 = \mu_0 H_0$;

$B_{iron} = \mu \mu_0 H_0$.

10.3: (a) $\oint \bar{H} d\bar{s} = NI$

Approximation: H-Field outside the coil ≈ 0, constant inside.

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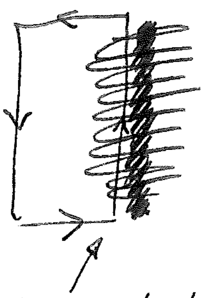


$$\Rightarrow \int \bar{H} d\bar{s} = Hl = NI \Rightarrow H = \frac{NI}{l}$$

$$\underline{B_{Air} = \mu_0 H, \quad B_{iron} = \mu \mu_0 H}$$

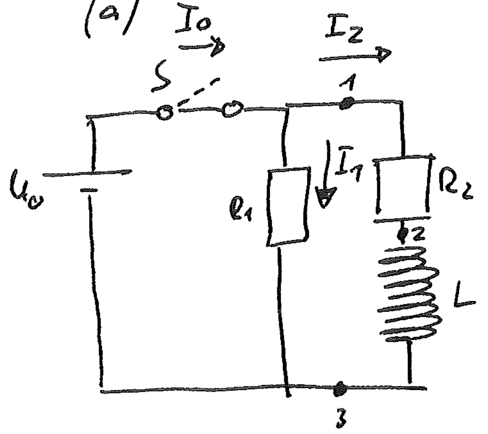
H is medium-independent
$\Rightarrow$ doesn't matter if inside or outside the iron

(b) $\Phi = LI = \int \bar{B} d\bar{A} = \int_{Air} B_{Air} d\bar{A} + \int_{Iron} B_{Iron} d\bar{A}$

$$= B_{Air} \pi \left(\frac{3}{4} r^2\right) + B_{Iron} \pi \left(\frac{1}{4} r^2\right)$$

$$= \underline{\pi r^2 \left(\frac{3}{4} + \frac{1}{4}\mu\right) \mu_0 \cdot H}$$

10.4. (a)



By the ~~[illegible]~~ mesh rule, $I_1 = \frac{U_0}{R_1}$ and $I_2 = \frac{U_0}{R_2}$.

(b) When we open the switch, we have a rapid change of I. Thus, a ~~current~~ voltage is induced in the coil that by Lenz's rule must try to continue the current that used to flow. (It will slowly cease, then, for obvious reasons) So the direction of I_2 stays the same, but the direction of I_1 is flipped.

(c) By continuity, directly after opening the switch $I_{induced}$ must equal I_2. It follows that $U_2' = I_2 \cdot R_2$, $U_1' = I_2 \cdot R_1$.

(d) The spark plug should be connected to 1 and 3. This is because then we can control both voltage and current over the spark plug by varying R_1 and R_2. Consequently, R_1 should be quite large compared to R_2 because then the voltage over the spark plug can be high enough to actually cause ignition.

(e) $U_{spark} = U_1'$; $U_1' = R_1 \cdot I_2 = R_1 \cdot \frac{U_0}{R_2} = \frac{R_1}{R_2} \cdot U_0 = 3000\,V$

The voltage is independent of L because no matter what inductance the coil has, it must always follow Lenz's rule and continue I. However, a large inductance lets the coil sustain the current for a longer time, which may be reasonable for a spark plug.