Experimental physics I, exercise sheet 10. Jannis Andrija Schnitzer and Pia Malika Renz

1	2	3	4
7	10	10	8

+7 10.1. a) ~~[illegible]~~

Bernoulli equation: $\frac{d}{dt}\left(p+\frac{1}{2}\rho v^2+\rho g h\right)=0$

p is pressure due to gravity, so $p=\rho g h$

$$\Rightarrow \frac{d}{dt}\left(\frac{1}{2}\rho v^2+2\rho g h\right)=0$$

$\frac{1}{2}$, 2, ρ and g are constant w.r.t. time.

v and h are functions of time

$$\Rightarrow \frac{1}{2}\cdot(v\cdot\dot v+\dot v\cdot v)+2\cdot g\cdot\dot h=0$$ +1

$$\Rightarrow v\cdot\dot v+2g\dot h=0$$

Change of volume: $\underset{\substack{\text{capital V}\\\text{(volume)}}}{\dot V}=\pi r^2\cdot\underset{\substack{\text{small v}\\\text{(velocity)}}}{v}=\pi R^2\cdot\dot h$

$$\Rightarrow \dot h=\frac{r^2}{R^2}\cdot\underset{\text{velocity}}{v}$$

$$\rightsquigarrow v\cdot\dot v+2\cdot g\cdot\frac{r^2}{R^2}\cdot v=0$$

$$\Rightarrow \text{either } v=0 \text{ or } \dot v=-2g\frac{r^2}{R^2}$$

$$\Rightarrow v(t)=\int-2g\frac{r^2}{R^2}\,dt=-2g\frac{r^2}{R^2}\cdot t \;+C$$

b) $V(t_0)=V_0-\int_0^{t_0}\dot V(t)\,dt$

$$=\pi R^2h_0-\int_0^{t_0}\pi r^2 v\,dt=\pi R^2h_0-\cancel{\pi}\,g\frac{r^4}{R^2}t_0^2$$ +2

$$V(t_0)=0\Rightarrow g\frac{r^4}{R^2}t_0^2=\pi R^2h_0$$

$$\Rightarrow t_0=\frac{R^2}{r^2}\cdot\sqrt{\frac{\pi\cdot h_0}{g}}$$

correct answer is $\frac{R^2}{r^2}\sqrt{\frac{2h_0}{g}}$ $3.14\approx 2$:)

c) Considering a single part of mass flowing out of the hole
For it holds true:

$$y(t)=y_0+v_0t+\frac{1}{2}gt^2 \quad \text{with the } y \text{ axis pointing downwards.}$$

$v_0\equiv v$, $y_0\equiv0\Rightarrow y(t)=vt+\frac{1}{2}gt^2$

$$\Rightarrow \frac{1}{2}gt^2+vt-y=0\Rightarrow t^2+\frac{2v}{g}-\frac{2y}{g}=0\Rightarrow t=\sqrt{\frac{v^2}{g^2}+\frac{2y}{g}}-\frac{v}{g}$$

Now considering the equation of continuity:

$$A_{hole}\cdot v = A_{jet,y}\cdot v_y$$

$$v_y = (v+gt) = \left(v + g\cdot\left(\sqrt{\frac{v^2}{g^2}+\frac{2y}{g}} - \frac{v}{g}\right)\right)$$

$$= v + \left(\sqrt{v^2+2yg} - v\right) = \sqrt{v^2+2yg}$$

$$\Rightarrow \pi r^2\cdot v = \pi r_y^2\cdot\sqrt{v^2+2yg}$$

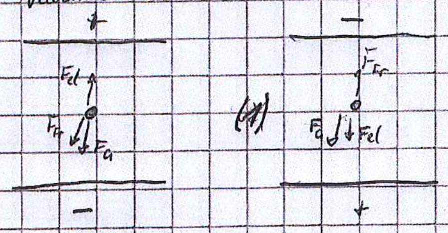
$$\Rightarrow r_y^2 = \frac{r^2 v}{\sqrt{v^2+2yg}} \qquad \Rightarrow r_y = r\cdot\frac{\sqrt{v}}{\sqrt[4]{v^2+2yg}}$$

+4

+10 10.2 a) Velocities are constant due to a friction force proportional to velocity.

(let v_2 be the upwards velocity, v_1 the downwards velocity)

sorry! → (2) (1)



$F_{el} = F_{Fr} + F_G$ $\qquad$ $F_{Fr} = F_{el} + F_G$

b) $\underbrace{F_{el}}_{\text{Electrical force}} = qE$

Gravity: $F_G = m\cdot g = \overset{\text{volume}}{V}\cdot\rho\cdot g = 4\pi r^3\rho g$ ← (Technically, a buoyancy force of $4\pi r^3\rho_{air}\cdot g$ has to be considered, but $\rho_{air} \ll \rho_{oil}$, so this can be neglected in our terms of accuracy)

Friction: $F_{Fr} = 6\pi\eta r\cdot \overset{\text{velocity}}{v}$

(1): $\frac{4}{3}\pi r^3\rho g + qE = 6\pi\eta r v_1$

(2)

$$\frac{4}{3}\pi r^3\rho g + 6\pi\eta r v_2 = qE \Leftrightarrow \frac{4}{3}\pi r^3\rho g - qE = -6\pi\eta r v_2$$

(1) + (2) yields $\frac{8}{3}\pi r^2\rho g = 6\pi\eta(v_1-v_2)$

$$\Rightarrow r^2 = \frac{9}{4}\frac{\eta(v_1-v_2)}{\rho g} \quad\Rightarrow r = \frac{3}{2}\sqrt{\frac{\eta(v_1-v_2)}{\rho g}}$$ +

(1) − (2) yields: $2qE = 6\pi\eta r(v_1+v_2)$

$$\Rightarrow q = \frac{3\pi\eta(v_1+v_2)}{E}\cdot r$$

Plugging in r: $q = \frac{3\pi\eta(v_1+v_2)}{E}\cdot\frac{3}{2}\sqrt{\frac{\eta(v_1-v_2)}{\rho g}}$

$$= \frac{9\pi}{2E}\sqrt{\frac{\eta^3(v_1-v_2)}{\rho g}}\cdot(v_1+v_2)$$ +

e) Plugging in values:

(Wolfram Alpha for calculation) $\Rightarrow q \approx 0{,}48 \cdot 10^{-19}\ C$

$$\frac{q}{e} \approx 3$$ (+)

10.3 a) $M_{max} = \frac{\pi R^4 G}{2L} \cdot \varphi_{max}$

+10

b) $P_{max} = \overline{M_{max} \cdot \omega} = M_{max} \cdot \omega$ because of parallelity.

c) $P_{max} = \frac{\pi R^4 G}{2L} \cdot \varphi_{max} \cdot \omega = \frac{\pi (1{,}5\,cm)^4\,(81\,GPa)}{2\cdot(2\,m)} \cdot \left(\frac{2{,}5}{360}\cdot 2\pi\right) \cdot \left(120 \cdot \frac{1}{s}\right)$

Wolfram Alpha for calculation

$= 16\,900\ W$ (+)

+8 10.4 a) Equation of movement:

$$M = M_R + M_B$$

$$I \cdot \ddot{\varphi} = -\alpha \cdot \varphi - \beta \cdot \dot{\varphi}$$ + (all M vectors point in z direction)

$$\Rightarrow \ddot{\varphi} + \frac{\beta}{I}\dot{\varphi} + \frac{\alpha}{I}\varphi = 0 \quad \Big| \text{ with } I = \frac{1}{2} m r^2$$ +

(Solution of differential equation $\ddot{z} + 2\gamma \dot{z} + \omega_0^2 z = 0$:

$$z(t) = e^{-\gamma t}\left(c_1 e^{i\omega_2 t} + c_2 e^{-i\omega_2 t}\right)$$ overdamped case only!!

With $\gamma = \frac{\beta}{2I}$ and $\omega_0^2 = \frac{\alpha}{I}$

($I = \frac{1}{2} m R^2$, so $\gamma = \frac{\beta}{mR^2}$).

b) For critical damping, $\gamma = \omega_0$, so $\frac{\beta}{mr^2} = \sqrt{\frac{2\alpha}{mr^2}}$

$$\Rightarrow \beta = \sqrt{2\alpha m r^2}$$

$$\underbrace{=}_{\text{numbers}} 0{,}0158\ \frac{kg\,m^2}{s}$$ +

c) For critical damping, we have the general form

$$\varphi(t) = (a + bt)e^{-\gamma t}$$ +

$$\dot{\varphi}(t) = -a\gamma e^{-\gamma t} + b e^{-\gamma t} - b\gamma t e^{-\gamma t}$$

γ_2 ?

Plugging in initial conditions:

$$\varphi(t=0) = a \overset{!}{=} \varphi_0 \Rightarrow a := \varphi_0$$

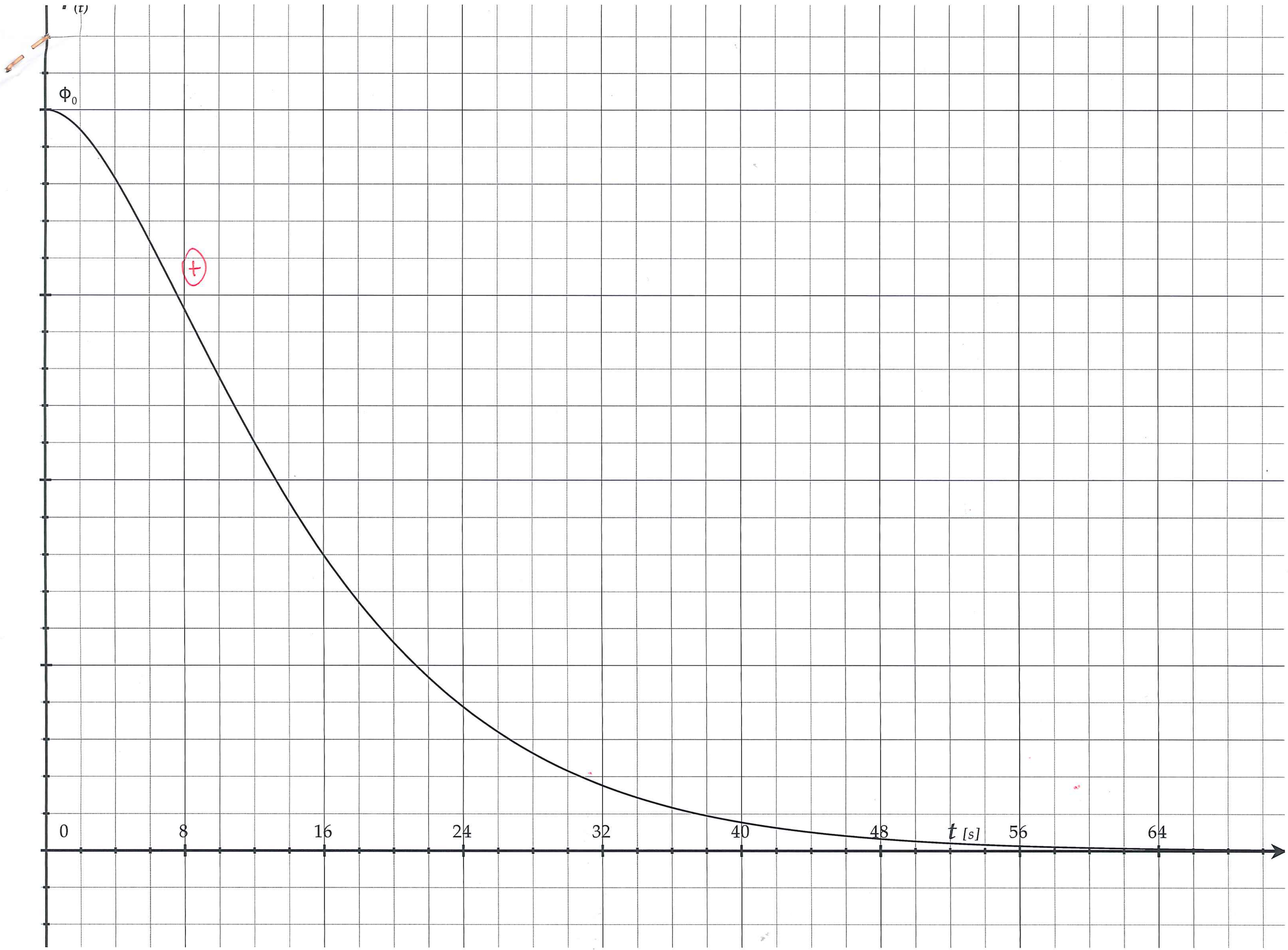
$$\dot{\varphi}(t=0) = -a\gamma + b \overset{!}{=} 0 \Rightarrow b := a\gamma = \varphi_0 \cdot \frac{\beta}{mr^2}$$

+8

$$\Rightarrow \varphi(t) = \varphi_0(1 + \gamma t) \cdot e^{-\gamma t} = \varphi_0\left(1 + \frac{\beta}{mr^2}\right) \cdot e^{-\frac{\beta t}{mr^2}}$$

d) Sketch: see extra sheet

e) $\varphi_0(1+\gamma t)e^{-\gamma t} = \varphi_0 \cdot 10^{-3}$ $\Rightarrow$ This can't be solved analytically; numeric solution (Wolfram Alpha): $t \approx 73$



(t)
Φ0
0
8
16
24
32
40
48
t [s]
56
64