MC: Differential- & Integralrechnung

- $\lim\limits_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} = \frac{df(x_0)}{dx} = f'(x_0)$

- Ableitungsregeln:
  - Potenzregel: $f(x) = x^n$; $f'(x) = \frac{1}{n} x^{n-1}$
  - Summenregel: $f(x) = g(x) + h(x)$; $f'(x) = g'(x) + h'(x)$
  - Faktorregel: $f(x) = c \cdot g(x)$; $f'(x) = c \cdot g'(x)$
    - special: $f(x) = \ln(x)$; $f'(x) = \frac{1}{x}$
  - Kettenregel: $f(g(x))' = f'(g(x)) \cdot g'(x)$
  - Produktregel: $(uv)' = u'v + uv'$
    - $(uvw)' = u'vw + uv'w + uvw'$
  - Ableitung d. Inversen: $f^{-1\prime}(x) = \frac{1}{f'(f^{-1}(x))}$

A1: $(\arcsin(x))'$; $y = \arcsin(x)$

$\sin y = \sin(\arcsin(x)) \Rightarrow x = \sin y$ (x)

mit $f^{-1\prime}(x) = \frac{1}{f'(f^{-1}(x))} = \frac{1}{(\sin y)'} = \frac{1}{\cos y}$ (i)

und $\sin^2 y + \cos^2 y = 1 \Rightarrow \cos y = \sqrt{1 - \sin^2 y}$ (ii)

(ii) in (i) ergibt $\frac{1}{\sqrt{1 - \sin^2 y}} \overset{(x)}{=} \frac{1}{\sqrt{1 - x^2}}$

- Integration: partielle Integration ≙ Produktintegration

$(uv)' = u'v + uv'$, integrieren: $\int (uv)' = \int u'v + \int uv'$

$uv = \int u'v + \int uv'$

$\int u'v = uv - \int uv'$

A2: $\int \sin x \cos x \, dx$ use: $u = \sin x$, $v' = \cos x$
$u' = \cos x$, $v = \sin x$

$\sin^2 x - \int \sin x \cos x \, dx = \int \sin x \cos x \, dx$

$\sin^2 x = 2 \int \sin x \cos x \, dx$

$\int \cos x \sin x = \frac{1}{2} \sin^2 x$

· Integration durch Substitution:

$$\underbrace{\int_a^b f(g(x)) \cdot g'(x)\, dx}_{} = \int_{g(a)}^{g(b)} f(z)\, dz$$

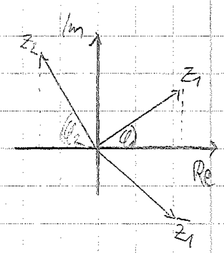
$$[F(g(x))]_a^b = F(g(b)) - F(g(a)) = [F(z)]_{g(a)}^{g(b)} \quad \text{mit } z = g(x)$$

A3: $\int_0^{0,5} \frac{1}{\sqrt{1-x^2}}\, dx = \int_0^{\frac{\pi}{6}} \frac{1}{\sqrt{1-\sin^2 t}} \cos t\, dt = \int_0^{\frac{\pi}{6}} \frac{1}{\cos t} \cdot \cos t\, dt = \int_0^{\frac{\pi}{6}} 1\, dt = \frac{\pi}{6}$

Ansatz: $g(t) = \sin(t) = x$

$\Rightarrow \frac{dx}{dt} = g'(t) = \cos(t) \Rightarrow dx = \cos(t) \cdot dt$

M 7: Komplexe Zahlen



· $\varphi = \arg(z_1) = \arctan\left(\frac{\mathrm{Im}(z_1)}{\mathrm{Re}(z_1)}\right)$; · komplexe Konjugation ≙ Spiegelung an Re

· $|z| = \sqrt{x^2 + y^2}$ $\qquad z := 1+i, \quad \bar{z} = 1-i$

· $e^{i\varphi} = \cos\varphi + i \sin\varphi$ und $e^{-i\varphi} = \cos\varphi - i\sin\varphi$

A4: $\frac{13-4i}{1-i} = \frac{(13-4i)(1+i)}{(1-i)(1+i)} = \frac{13-4i+13i+4}{2} = \underbrace{\frac{9i}{2}}_{\mathrm{Im}} + \underbrace{\frac{17}{2}}_{\mathrm{Re}}$

Kinematik d. freien Massenpunktes

1) 1-D Bewegung

$a(t) = \frac{dv(t)}{dt} = \ddot{x}$, $v(t) = \frac{dx(t)}{dt} = \dot{x} = v_0 + \int_{t_0}^{t} \ddot{x}\, dt$

$x(t) = x_0 + v_0 t + \frac{a_0}{2} t^2 \rightarrow v(t) = \dot{x}(t) = v_0 + a_0 t \rightarrow a(t) = \dot{v}(t) = \ddot{x}(t) = a_0$

2) Mehrdimensionale Bew.

· „Trajektorie": Bahnkurve eines Teilchens, $r(t) = \sum_{j=1}^{3} x_j(t) e_j \equiv (x_1(t), x_2(t), x_3(t))$
bzw. $(t, \vec{x}(t))$ mit $x \in \mathbb{R}^4$

· vollständige Beschreibung braucht $\vec{a} = \dot{\vec{v}} = \ddot{\vec{x}}$

Bsp: Schraube:
$\vec{x} = \begin{pmatrix} R\cos(\omega t) \\ R\sin(\omega t) \\ v_0 t \end{pmatrix}$; $\dot{\vec{x}} = \vec{v} = \begin{pmatrix} -R\omega\sin(\omega t) \\ R\omega\cos(\omega t) \\ v_0 \end{pmatrix}$, $\ddot{\vec{x}} = \dot{\vec{v}} = \vec{a} = -R\omega^2 \begin{pmatrix} \cos(\omega t) \\ \sin(\omega t) \\ 0 \end{pmatrix}$

· Bogenlänge: $s(t) = \int |d\vec{x}| = \int_{t_0}^{t} dt' \left|\frac{d\vec{x}}{dt'}\right| = \int_{t_0}^{t} dt' \sqrt{\dot{\vec{x}}(t')^2} = \int_{t_0}^{t} dt' \sqrt{\vec{v}(t')^2}$

Anm.: $\frac{ds(t)}{dt} = \left|\frac{d\vec{x}}{dt}\right| = |\vec{v}|$

→ · Tangentenvektor $\vec{T}(s) = \frac{d\vec{x}}{ds}$ mit $\vec{T} \parallel \vec{v}$ und $|\vec{T}(s)| = \left|\frac{d\vec{x}}{ds}\right| = \left|\frac{dx(t(s))}{|\vec{v}|\, dt}\right| = \left|\frac{\vec{v}\, dt}{|\vec{v}|\, dt}\right| = 1$

Anm.: $\frac{d\vec{T}}{ds} \perp \vec{T}$ wegen $0 = \frac{d}{ds}(1) = \frac{d}{ds}(\vec{T}\cdot\vec{T}) = \frac{d\vec{T}}{ds}\vec{T} + \frac{d\vec{T}}{ds}\vec{T} = 2\vec{T}\frac{d\vec{T}}{ds}$

· Krümmungsradius: $\rho = \frac{1}{\left|\frac{d\vec{T}}{ds}\right|}$ (Radius d. Schmiegekreises)

→ · Hauptnormalenvektor: $\vec{N} = \frac{d\vec{T}/ds}{|d\vec{T}/ds|} = \rho \frac{d\vec{T}}{ds}$

→ · Binormalenvektor: $\vec{B} = \vec{T} \times \vec{N}$