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16.35 Uhr
Gleis 5

1a)
$v_1 = (1, 2, 4)$; $v_2 = (1, 0, -1)$; $v_3 = (0, 1, 2)$

$$\alpha_1 \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} + \alpha_2 \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + \alpha_3 \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} = 0$$

$$\left.\begin{array}{lll} \alpha_1 + \alpha_2 = 0 & \Rightarrow \alpha_1 = -\alpha_2 & \Rightarrow \alpha_2 = 0 \\ 2\alpha_1 + \alpha_3 = 0 & \alpha_1 = -\frac{1}{2}\alpha_3 & \Rightarrow \alpha_3 = 0 \\ 4\alpha_1 - \cancel{1}\alpha_2 + 2\alpha_3 = 0 & \Rightarrow 4\alpha_1 + \alpha_1 \cancel{*} - 4\alpha_1 = 0 & \\ & \Rightarrow \alpha_1 = 0 & \end{array}\right\} \text{l.u.}$$

Basis im $\mathbb{R}^3$, wegen $\dim(\mathrm{Lin}(v_1, v_2, v_3)) = 3$ und $\dim(\mathbb{R}^3) = 3$

[2.2. $\hat{e}_1, \hat{e}_2, \hat{e}_3 \in \mathrm{Lin}(v_1, v_2, v_3)$ mit $\hat{e}_{ij} = \delta_{ij}$]

$$\begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} = \alpha_1 \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} + \alpha_2 \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + \alpha_3 \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$

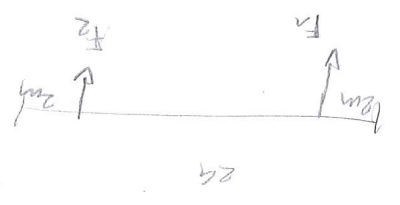
$1 = \alpha_1 + \alpha_2$
$1 = 2\alpha_1 + \alpha_3$
$3 = 4\alpha_1 - \alpha_2 + 2\alpha_3$

$1 = 2\alpha_1 + \alpha_3$ $\quad \alpha_1 = 2$
$4 = 5\alpha_1 + 2\alpha_3$ $\quad \alpha_2 = -1$

$-2 = 4\alpha_1 - 2\alpha_3$ $\quad \alpha_3 = -3$
$4 = 5\alpha_1 + 2\alpha_3$

$2 = \alpha_1$
$-1 = \alpha_2$
$-3 = \alpha_3$

$L = 24\,m$; $F_1 = 5 \cdot 10^3\,N$ $\quad F_2 = 3 \cdot 10^3\,N$

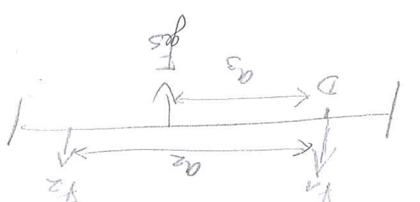


$$|\vec{F_1}| \cdot L_1 = |\vec{F_2}|\, L_2$$

Hebelgesetz : linker Block als Auflagepunkt

center of rotation : e.g. ~~left end~~ left point of support :

F_s.



$$F_1 \cdot a_2 = F_{ges} \cdot a_3 \Rightarrow a_3 = \frac{F_1 \cdot a_2}{F_{ges}} = \frac{3{,}0\,m \cdot 20\,kN \cdot m}{8{,}0\,kN} = 7{,}5\,m$$

$d = 2m + a_3 = 9{,}5\,m$

b)

1b)

(v_1, v_2, v_3) ; (v_1^*, v_2^*, v_3^*) ; $(\mathbb{R}^3)^* = \mathrm{Hom}_{\mathbb{R}}(\mathbb{R}^3, \mathbb{R})$ $\qquad$ $\varphi: \mathbb{R}^3 \to \mathbb{R}$, $(x,y,z) \mapsto -2x - 3y + 4z$ linear

def. durch $v_i^*(v_j) = \delta_{ij}$ mit $v_i^* \in (\mathbb{R}^3)^*$

$$\left.\begin{array}{lll} v_1^*(v_1) = 1 & v_1^*(v_2) = 0 & v_1^*(v_3) = 0 \\ v_2^*(v_1) = 0 & v_2^*(v_2) = 1 & v_2^*(v_3) = 0 \\ v_3^*(v_1) = 0 & v_3^*(v_2) = 0 & v_3^*(v_3) = 1 \end{array}\right\} \text{ wegen } v_i^*(v_j) = \delta_{ij}$$

und

$v_1^*(x,y,z) = \alpha_1 x + \beta_1 y + \gamma_1 z$ $\qquad$ $v_1 = (1,2,4)$

$v_2^*(x,y,z) = \alpha_2 x + \beta_2 y + \gamma_2 z$ $\qquad$ $v_2 = (1,0,-1)$

$v_3^*(x,y,z) = \alpha_3 x + \beta_3 y + \gamma_3 z$ $\qquad$ $v_3 = (0,1,2)$

LGS:

$v_1^*(v_1) = \alpha_1 + 2\beta_1 + 4\gamma_1 = 1 \Rightarrow \gamma_1 - 4\gamma_1 + 4\gamma_1 = 1 \Rightarrow \gamma_1 = 1$

$\quad (v_2) = \alpha_1 - \gamma_1 = 0 \Rightarrow \alpha_1 = \gamma_1 \Rightarrow \alpha_1 = 1$ $\qquad$ $\boxed{1, -2, 1}$

$\quad (v_3) = \beta_1 + 2\gamma_1 = 0 \Rightarrow \beta_1 = -2\gamma_1 \Rightarrow \beta_1 = -2$

$v_2^*(v_1) = \alpha_2 + 2\beta_2 + 4\gamma_2 = 0 \Rightarrow 1 + \gamma_2 - 4\gamma_2 + 4\gamma_2 = 0 \Rightarrow \gamma_2 = -1$

$\quad (v_2) = \alpha_2 - \gamma_2 = 1 \Rightarrow \alpha_2 = 1 + \gamma_2 \Rightarrow \alpha_2 = 0$

$\quad (v_3) = \beta_2 + 2\gamma_2 = 0 \Rightarrow \beta_2 = -2\gamma_2 \Rightarrow \beta_2 = 2$ $\qquad$ $\boxed{0, 2, -1}$

$v_3^*(v_1) = \alpha_3 + 2\beta_3 + 4\gamma_3 = 0 \Rightarrow \gamma_3 + 2 - 4\gamma_3 + 4\gamma_3 = 0 \Rightarrow \gamma_3 = -2$

$\quad (v_2) = \alpha_3 - \gamma_3 = 0 \Rightarrow \alpha_3 = \gamma_3 \Rightarrow \alpha_3 = -2$

$\quad (v_3) = \beta_3 + 2\gamma_3 = 1 \Rightarrow \beta_3 = 1 - 2\gamma_3 \Rightarrow \beta_3 = 5$ $\qquad$ $\boxed{-2, 5, -2}$

$\Rightarrow$ duale Basis:

$v_1^*(x,y,z) = x - 2y + z$

$v_2^*(x,y,z) = 2y - z$

$v_3^*(x,y,z) = -2x + 5y - 2z$; φ als Linearkombination:

$$-2x - 3y + 4z = r(x - 2y + z) + s(2y - z) + t(-2x + 5y - 2z)$$
$$= x(r - 2t) + y(-2r + 2s + 5t) + z(r - s - 2t)$$

$\Rightarrow r - 2t = -2$

$-2r + 2s + 5t = -3$ $\quad$ I +

$2r - 2s - 22t = 4$ $\quad$ I · 2

$3t = -1$

$t = -\frac{1}{3} \Rightarrow r = -2,\bar{6} = -2\frac{2}{3} = -\frac{8}{3}$

$\Rightarrow s = -\frac{20}{3} / 2 = -\frac{10}{3}$

$\Rightarrow \varphi = -\frac{8}{3} v_1^* - \frac{10}{3} v_2^* - \frac{1}{3} v_3^*$

2a) $u_1=(1,3)$; $u_2=(2,1)$; $u_3=(4,7)$

Basis ist bspw. $\hat{e}_1=\begin{pmatrix}0\\1\end{pmatrix}$; $\hat{e}_2=\begin{pmatrix}1\\0\end{pmatrix}$

$-5\cdot\begin{pmatrix}0\\1\end{pmatrix}=2\cdot\begin{pmatrix}2\\1\end{pmatrix}-\begin{pmatrix}4\\7\end{pmatrix}$

$\begin{pmatrix}0\\-5\end{pmatrix}=\begin{pmatrix}4\\2\end{pmatrix}-\begin{pmatrix}4\\7\end{pmatrix}$

$-5\cdot\hat{e}_2=-5\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}1\\3\end{pmatrix}+-3\begin{pmatrix}2\\1\end{pmatrix}$

$\begin{pmatrix}-5\\0\end{pmatrix}=\begin{pmatrix}1\\3\end{pmatrix}-\begin{pmatrix}6\\3\end{pmatrix}$

$\alpha_1\hat{e}_1=\alpha_2u_2+\alpha_3u_3$

mit $\alpha_1=-5$, $\alpha_2=2$, $\alpha_3=-1$

$\beta_1\hat{e}_2=\beta_2u_1+\beta_3u_2$

mit $\beta_1=-5$, $\beta_2=1$, $\beta_3=-3$

$\Rightarrow$ Basis lässt sich aus den Vektoren des Systems (u_1,u_2,u_3) erzeugen $\Rightarrow$ (u_1,u_2,u_3) ist Erzeugendensystem

b) $f:\mathbb{R}^2\to\mathbb{R}^2$; $f(u_1)=(-2,-1)$; $f(u_2)=(6,3)$; $f(u_3)=(2,1)$

$\lambda(x,y)\longmapsto\lambda(x',y')$

mit $(\lambda x,\lambda y)\longmapsto(\lambda x',\lambda y')$ gilt: $x'=\alpha_1x+\alpha_2y$, $y'=\alpha_3x+\alpha_4y$

$\Rightarrow$ $-2=\alpha_1+3\alpha_2$, $6=2\alpha_1+\alpha_2$, $2=4\alpha_1+7\alpha_2$ $\quad\hookrightarrow$ $\alpha_1=4$, $\alpha_2=\cancel{3}-2$

und $-1=\alpha_3+\alpha_4 3$, $3=2\alpha_3+\alpha_4$, $1=4\alpha_3+7\alpha_4$ $\quad\hookrightarrow$ $\alpha_3=2$, $\alpha_4=-1$

$\Rightarrow$ $\left.\begin{array}{l}x'=4x-2y\\ y'=2x-y\end{array}\right\}\Rightarrow f(4x-2y,2x-y)$

c) $g:\mathbb{R}^2\to\mathbb{R}^2$; $g(u_1)=(-2,-1)$; $g(u_2)=(6,3)$; $g(u_3)=(1,1)$

Da $u_3=2\cdot u_1+u_2$ Linearkombination und *(Annahme: Es gebe g mit den geg. Punkten,) die Linearität fordert $g(u_3)=2g(u_1)+g(u_2)$

$\begin{pmatrix}1\\1\end{pmatrix}=2\begin{pmatrix}-2\\-1\end{pmatrix}+\begin{pmatrix}6\\3\end{pmatrix}=\begin{pmatrix}2\\1\end{pmatrix}$ ↯ $\Rightarrow\nexists g$

d)

- $\operatorname{im}(f)=\{(4x-2y,2x-y)\mid x,y\in\mathbb{R}\}$

Sei $\alpha=2x-y$ $\Rightarrow$ $\operatorname{im}(f)=\{(2\alpha,\alpha)\mid\alpha\in\mathbb{R}\}$

$\Rightarrow\operatorname{im}(f)=\{\alpha(2,1)\mid\alpha\in\mathbb{R}\}\Rightarrow\operatorname{im}(f)=\operatorname{Lin}((2,1))$

$\Rightarrow b=(2,1)$

- $\ker(f)=\{(x,y)\mid f(x,y)=0\}$ mit $4x-2y=0\ \wedge\ 2x-y=0$

$4x-4x=0 \quad\sim\quad 2x=y$

$4x=4x$

$0=0$

$=\{(x,y)\mid 2x=y\}$

$=\{(x,2x)\mid x\in\mathbb{R}\}=\{(1,2)\mid x\in\mathbb{R}\}$

$=\operatorname{Lin}((1,2))\Rightarrow b=(1,2)$

~~dim(ker(f)) = Rg~~

$\dim(\operatorname{im}(f))=\operatorname{Rg}(f)=1$