Versuch 255: Röntgenspektrometer

```
Needs["ErrorBarPlots`"]
```

Messung des Röntgenspektrums mit einem LiF-Kristall

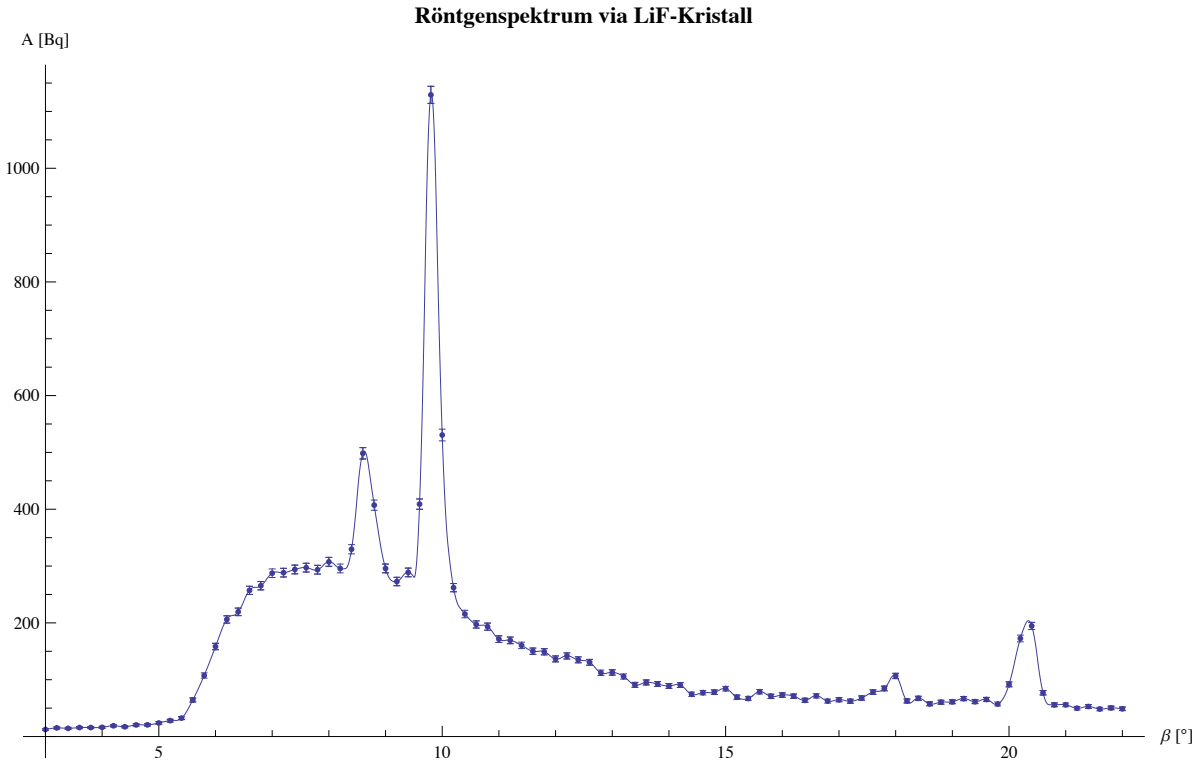
```
spectrum_LiF = Import[
  "/Users/jannis/Dropbox/uniself/AP2/2.2/255 Röntgenspektrometer/1a/data.dat"]
```

```
{{3., 12.4}, {3.2, 15.2}, {3.4, 14.4}, {3.6, 15.8}, {3.8, 15.8}, {4., 16.2},
 {4.2, 19.}, {4.4, 17.2}, {4.6, 20.4}, {4.8, 20.6}, {5., 23.8}, {5.2, 28.},
 {5.4, 32.4}, {5.6, 64.4}, {5.8, 107.4}, {6., 158.4}, {6.2, 206.}, {6.4, 219.4},
 {6.6, 257.2}, {6.8, 265.4}, {7., 287.4}, {7.2, 288.4}, {7.4, 294.},
 {7.6, 297.2}, {7.8, 293.6}, {8., 307.4}, {8.2, 295.8}, {8.4, 329.6},
 {8.6, 498.4}, {8.8, 407.2}, {9., 295.8}, {9.2, 272.8}, {9.4, 288.8},
 {9.6, 409.}, {9.8, 1129.2}, {10., 530.6}, {10.2, 262.}, {10.4, 215.6},
 {10.6, 197.2}, {10.8, 193.4}, {11., 171.4}, {11.2, 169.2}, {11.4, 160.4},
 {11.6, 150.2}, {11.8, 149.2}, {12., 136.6}, {12.2, 141.8}, {12.4, 135.},
 {12.6, 130.6}, {12.8, 112.2}, {13., 112.8}, {13.2, 105.6}, {13.4, 91.},
 {13.6, 95.2}, {13.8, 92.4}, {14., 89.}, {14.2, 90.6}, {14.4, 74.4}, {14.6, 77.2},
 {14.8, 78.}, {15., 84.2}, {15.2, 69.2}, {15.4, 67.}, {15.6, 78.8}, {15.8, 70.8},
 {16., 73.}, {16.2, 71.}, {16.4, 63.8}, {16.6, 71.6}, {16.8, 62.4}, {17., 64.8},
 {17.2, 62.2}, {17.4, 67.8}, {17.6, 78.2}, {17.8, 84.6}, {18., 106.6},
 {18.2, 62.6}, {18.4, 67.2}, {18.6, 57.6}, {18.8, 60.4}, {19., 61.2},
 {19.2, 66.8}, {19.4, 61.4}, {19.6, 65.2}, {19.8, 57.2}, {20., 91.8},
 {20.2, 172.6}, {20.4, 194.6}, {20.6, 77.}, {20.8, 55.8}, {21., 55.8},
 {21.2, 49.8}, {21.4, 52.8}, {21.6, 48.4}, {21.8, 50.4}, {22., 48.8}}
```

```
t_tor,a = Quantity[5, "Seconds"]
```

```
5 s
```

```
plotLiF = With[{d = spectrumLiF}, ErrorListPlot[
    Table[{v, ErrorBar[Sqrt[v[[2]] / QuantityMagnitude[ttor,a]]]}, {v, d}],
    Joined → True, InterpolationOrder → 2, Mesh → Full, PlotRange → All,
    ImageSize → Full, AxesOrigin → {d[[1, 1]], 0}, AxesLabel → {"β [°]", "A [Bq]"},
    PlotLabel → Style["Röntgenspektrum via LiF-Kristall", Bold]]]
```



Extrapolation des Grenzwinkels

```
lpLiF = Select[spectrumLiF, 5.6 ≤ #[[1]] ≤ 6.2 &]
```

```
{{5.6, 64.4}, {5.8, 107.4}, {6., 158.4}, {6.2, 206.}}
```

```
extrapolationLiF = LinearModelFit[lpLiF, x, x,
  VarianceEstimatorFunction → (1 &), Weights → Sqrt[QuantityMagnitude[ttor,a]]^2 / Sqrt[lpLiF[[All, 2]]]^2]
```

```
FittedModel[-1256.98 + 235.763 x]
```

```
extrapolationLiF["ParameterTable"]
```

	Estimate	Standard Error	t-Statistic	P-Value
1	−1256.98	63.9282	−19.6624	0.00257659
x	235.763	11.0094	21.4147	0.00217349

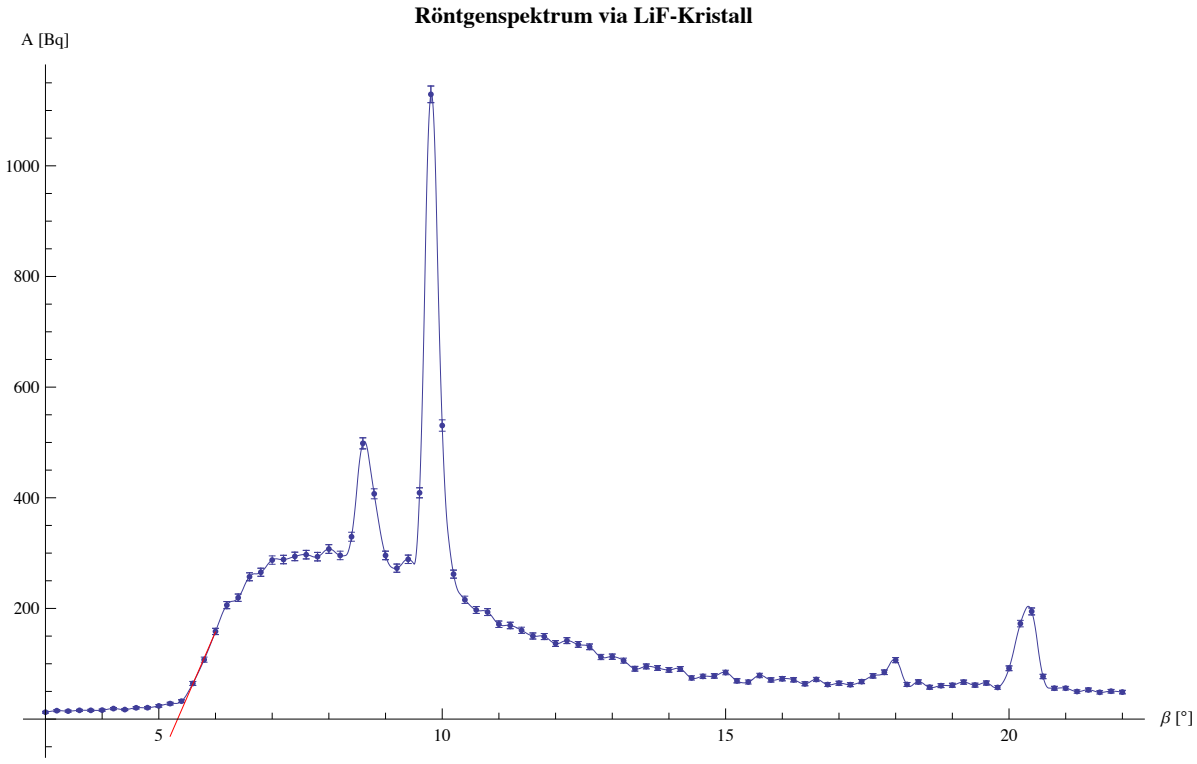
```
{c, Δc} = extrapolationLiF["ParameterTableEntries"][[1, 1 ;; 2]]
```

```
{-1256.98, 63.9282}
```

```
{m, Δm} = extrapolationLiF["ParameterTableEntries"][[2, 1 ;; 2]]
```

```
{235.763, 11.0094}
```

```
Show[plot_LiF, Plot[extrapolation_LiF[x], {x, 5.2, 6}, PlotStyle → Red]]
```



$$\beta_{grenz,LiF} = -\frac{c}{m}\,°$$

0.093053

$$\Delta\beta_{grenz,LiF} = \text{Sqrt}\left[\left(\frac{c\,\Delta m}{m^2}\right)^2 + \left(\frac{\Delta c}{m}\right)^2\right]°$$

0.00642483

$$\text{PlusMinus}\left[\text{NumberForm}\left[\frac{360}{2\,\pi}\,\beta_{grenz,LiF},\,2\right],\,\text{NumberForm}\left[\frac{360}{2\,\pi}\,\Delta\beta_{grenz,LiF},\,1\right]\right]$$

5.3 ± 0.4

$$d_{LiF} = \text{Quantity}[201.4,\,\text{"Picometers"}]$$

201.4 pm

$$\lambda_{LiF}[\beta\_,\,\Delta\beta\_,\,n\_:1] := \left\{\frac{2}{n}\,d_{LiF}\,\text{Sin}[\beta],\,\frac{2}{n}\,d_{LiF}\,\text{Cos}[\beta]\,\Delta\beta\right\}$$

$$\left\{\lambda_{grenz,LiF},\,\Delta\lambda_{grenz,LiF}\right\} = \lambda_{LiF}\left[\beta_{grenz,LiF},\,\Delta\beta_{grenz,LiF}\right]$$

{37.4277 pm, 2.57672 pm}

$$\frac{\left\{\lambda_{grenz,LiF},\,\Delta\lambda_{grenz,LiF}\right\}\,\text{Quantity}[30,\,\text{"Kiloelectronvolts"}]}{\text{Quantity}[\text{"SpeedOfLight"}]}\;//\;\text{UnitConvert}$$

$\left\{6.00073\times10^{-34}\ \text{kg m}^2/\text{s},\ 4.13122\times10^{-35}\ \text{kg m}^2/\text{s}\right\}$

```
Quantity["PlanckConstant"] // UnitConvert
```

$6.626070 \times 10^{-34}\ \mathrm{kg\,m^2/s}$

Beginn der 2. Ordnung:

$$\left\{\frac{360}{2\pi}\,\mathrm{ArcSin}\left[\frac{\lambda_{\mathrm{grenz,LiF}}}{d_{\mathrm{LiF}}}\right],\ \frac{360}{2\pi}\,\frac{\Delta\lambda_{\mathrm{grenz,LiF}}}{d_{\mathrm{LiF}}}\,\frac{1}{\sqrt{1-\left(\frac{\lambda_{\mathrm{grenz,LiF}}}{d_{\mathrm{LiF}}}\right)^2}}\right\}$$

```
{10.71, 0.746041}
```

Vermessung der K-Linien

```
t_tor,b = Quantity[20, "Seconds"]
```

20 s

K_α-Linie 1. Ordnung

```
data_LiF,Kα,1 = Import["/Users/jannis/Dropbox/uniself/AP2/2.2/255
    Röntgenspektrometer/1b/1alpha/data.dat"]
```

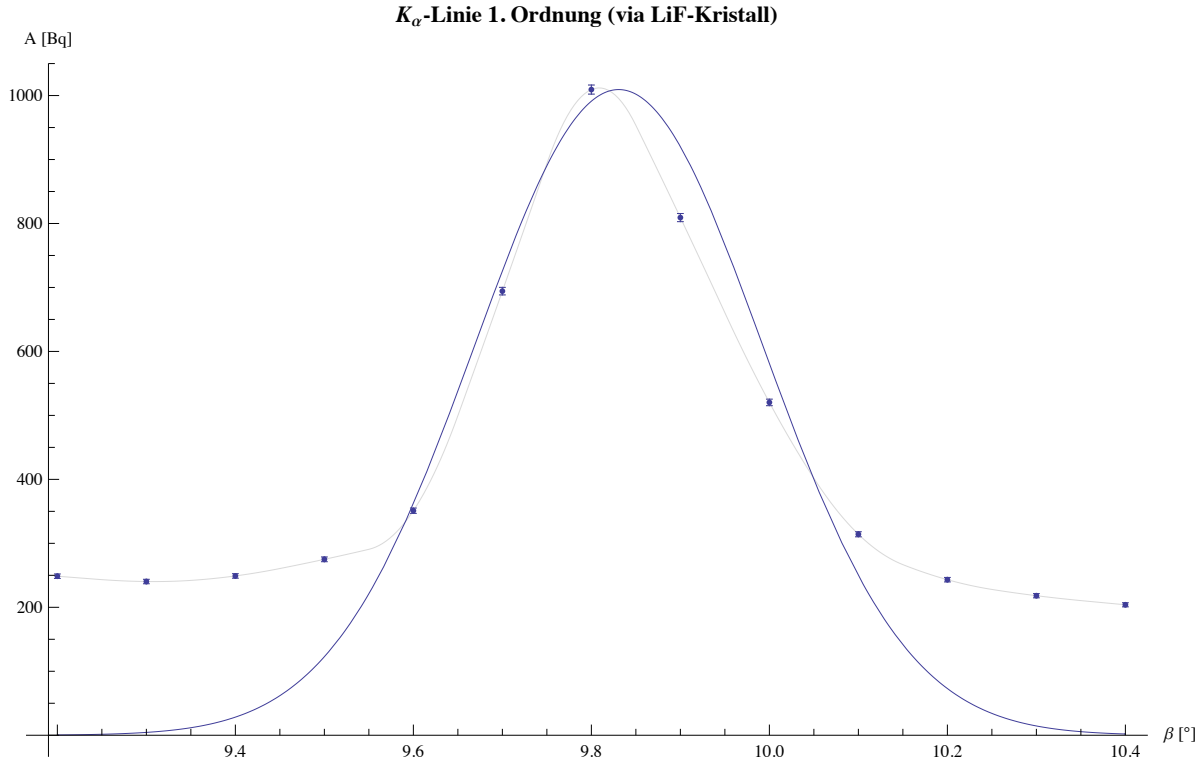
```
{{9.2, 248.6}, {9.3, 240.3}, {9.4, 249.05}, {9.5, 275.},
 {9.6, 351.}, {9.7, 694.25}, {9.8, 1009.35}, {9.9, 809.25}, {10., 520.3},
 {10.1, 314.2}, {10.2, 243.2}, {10.3, 218.15}, {10.4, 203.95}}
```

```
plot_LiF,Kα,1 = With[{d = data_LiF,Kα,1}, ErrorListPlot[
    Table[{v, ErrorBar[Sqrt[v[[2]] / QuantityMagnitude[t_tor,b]]]}, {v, d}],
    Joined → True, InterpolationOrder → 2, PlotStyle → LightGray,
    MeshStyle → ColorData[1, 1], Mesh → Full, PlotRange → All, ImageSize → Full,
    AxesOrigin → {d[[1, 1]] - .01, 0}, AxesLabel → {"β [°]", "A [Bq]"},
    PlotLabel → Style["Kα-Linie 1. Ordnung (via LiF-Kristall)", Bold]]];
```

```
fit_LiF,Kα,1 = With[{d = Select[data_LiF,Kα,1, 9.6 ≤ #[[1]] ≤ 10.1 &]},
  NonlinearModelFit[d, Max[d[[All, 2]]] Exp[-1/2 (x - μ)^2/σ^2],
   {{μ, Mean[d[[All, 1]]]}, {σ, StandardDeviation[d[[All, 1]]]}}, {x},
   Weights → Sqrt[QuantityMagnitude[t_tor,b]]^2/Sqrt[d[[All, 2]]]^2, VarianceEstimatorFunction → (1 &)]]
```

```
FittedModel[1009.35 e^(-19.2619 ≪1≫^2)]
```

```
Show[plot_{LiF,Kα,1}, Plot[fit_{LiF,Kα,1}[x], {x, 9.2, 10.4}]]
```




```
fit_{LiF,Kα,1}["ParameterTable"]
```

	Estimate	Standard Error	t-Statistic	P-Value
μ	9.83072	0.000686895	14311.8	1.43012×10^{-16}
σ	0.161115	0.00055241	291.658	8.29133×10^{-10}

```
{λ_{Kα,1}, Δλ_{Kα,1,fit}} = λ_{LiF} @@ (fit_{LiF,Kα,1}["ParameterTableEntries"][[1, 1 ;; 2]] °)
```

```
{68.7732 pm, 0.0047581 pm}
```

FWHM (mit Fit-Fehler):

```
{Δλ_{Kα,1}, Δλ_{Kα,1,Δfit}} =
 2 √(2 Log[2]) λ_{LiF} @@ (fit_{LiF,Kα,1}["ParameterTableEntries"][[2, 1 ;; 2]] °)
```

```
{2.66722 pm, 0.00914502 pm}
```

K_β-Linie 1. Ordnung

```
data_{LiF,Kβ,1} = Import["/Users/jannis/Dropbox/uniself/AP2/2.2/255
    Röntgenspektrometer/1b/1beta/data.dat"]
```

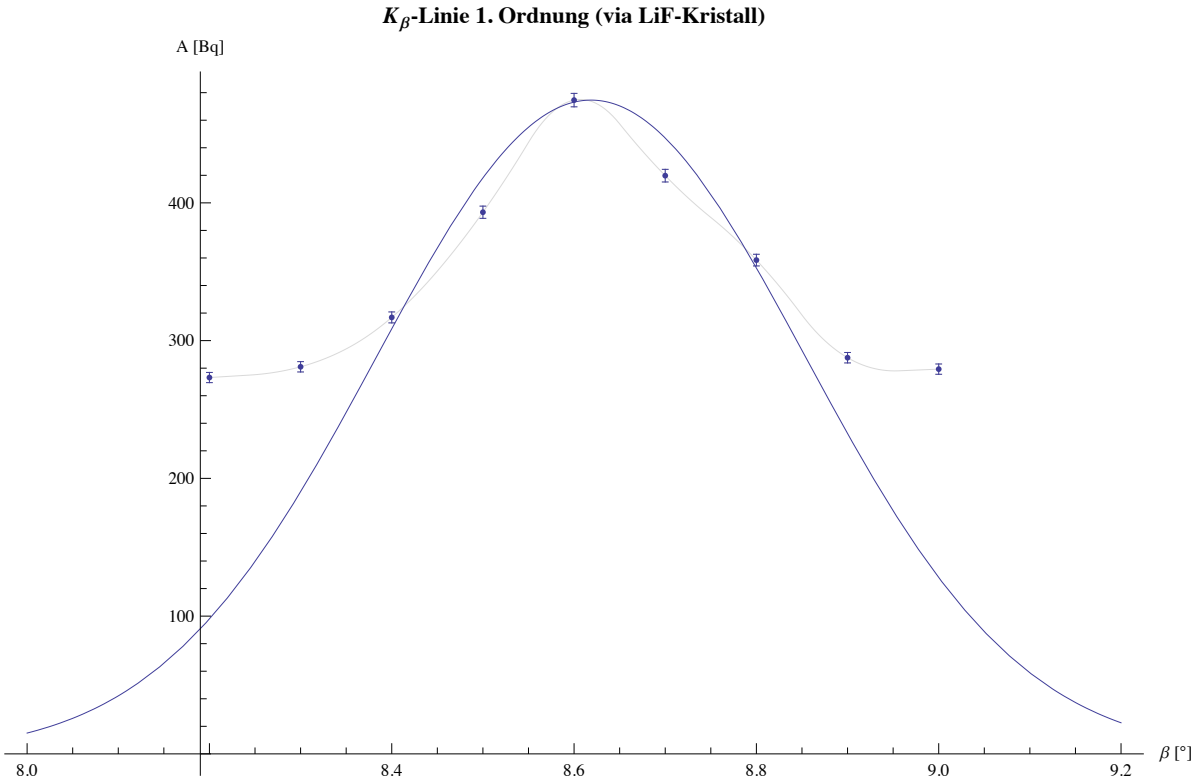
```
{{8.2, 273.2}, {8.3, 281.}, {8.4, 316.85}, {8.5, 393.2},
 {8.6, 474.6}, {8.7, 419.75}, {8.8, 358.45}, {8.9, 287.6}, {9., 279.3}}
```

```
plot_{LiF,Kβ,1} = With[{d = data_{LiF,Kβ,1}}, ErrorListPlot[
    Table[{v, ErrorBar[Sqrt[v[[2]] / QuantityMagnitude[t_{tor,b}]]]}, {v, d}],
    Joined → True, InterpolationOrder → 2, PlotStyle → LightGray,
    MeshStyle → ColorData[1, 1], Mesh → Full, PlotRange → All, ImageSize → Full,
    AxesOrigin → {d[[1, 1]] - .01, 0}, AxesLabel → {"β [°]", "A [Bq]"},
    PlotLabel → Style["Kβ-Linie 1. Ordnung (via LiF-Kristall)", Bold]]];
```

```
fit_{LiF,Kβ,1} = With[{d = Select[data_{LiF,Kβ,1}, 8.4 ≤ #[[1]] ≤ 8.8 &]},
  NonlinearModelFit[d, Max[d[[All, 2]]] Exp[-1/2 (x - μ)^2/σ^2],
   {{μ, Mean[d[[All, 1]]]}, {σ, StandardDeviation[d[[All, 1]]]}}, {x},
   VarianceEstimatorFunction → (1 &), Weights → Sqrt[QuantityMagnitude[t_{tor,b}]]^2/Sqrt[d[[All, 2]]]^2]]
```

FittedModel[474.6 $e^{-9.0083 \ll 1 \gg^2}$]

```
Show[plot_{LiF,Kβ,1}, Plot[fit_{LiF,Kβ,1}[x], {x, 8, 9.2}]]
```




```
fit_{LiF,Kβ,1}["ParameterTable"]
```

	Estimate	Standard Error	t-Statistic	P-Value
μ	8.61857	0.00215054	4007.63	3.42617×10^{-11}
σ	0.235594	0.002784	84.6241	3.63723×10^{-6}

```
{λ_{Kβ,1}, Δλ_{Kβ,1,fit}} = λ_{LiF} @@ (fit_{LiF,Kβ,1}["ParameterTableEntries"][[1, 1 ;; 2]] °)
```

{60.3619 pm, 0.014948 pm}

FWHM (mit Fit-Fehler):

```
{Δλ_{Kβ,1}, Δλ_{Kβ,1,Δfit}} =
 2 Sqrt[2 Log[2]] λ_{LiF} @@ (fit_{LiF,Kβ,1}["ParameterTableEntries"][[2, 1 ;; 2]] °)
```

{3.9002 pm, 0.0460883 pm}

K_α-Linie 2. Ordnung

```
data_LiF,Kα,2 = Import["/Users/jannis/Dropbox/uniself/AP2/2.2/255
    Röntgenspektrometer/1b/2alpha/data.dat"]
```

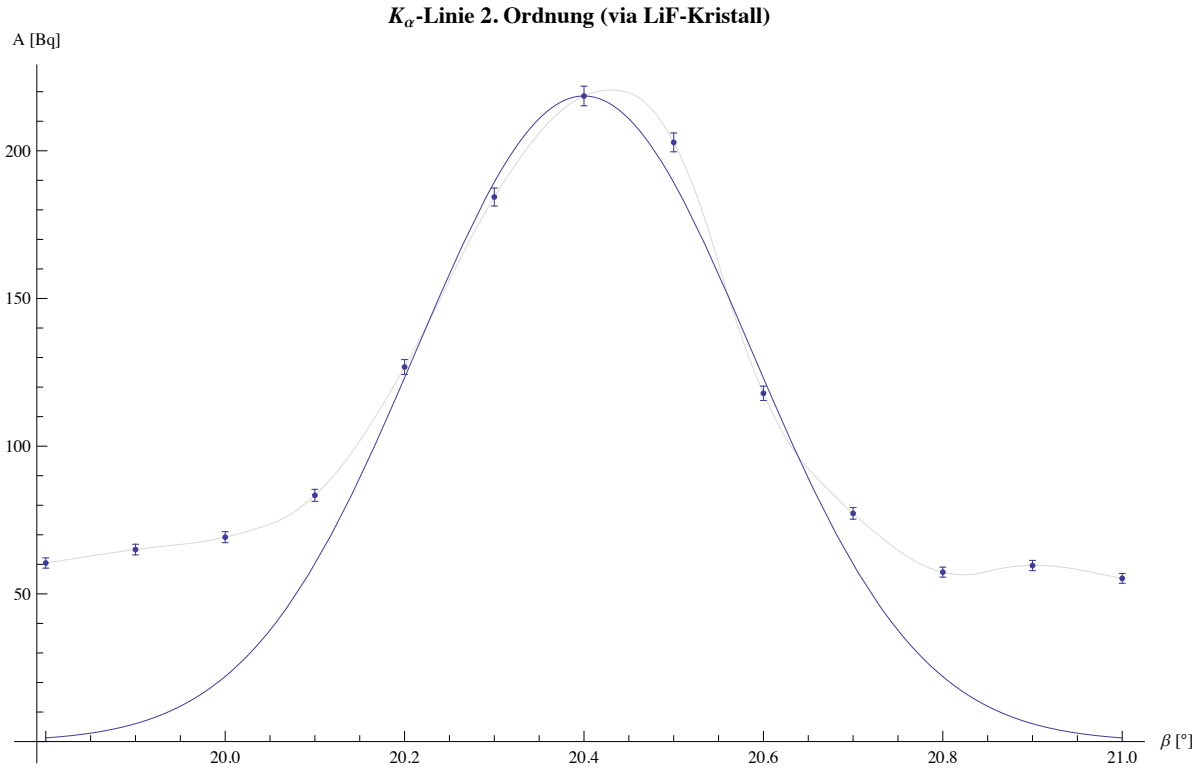
```
{{19.8, 60.45}, {19.9, 65.}, {20., 69.2}, {20.1, 83.35},
 {20.2, 126.8}, {20.3, 184.35}, {20.4, 218.55}, {20.5, 202.85},
 {20.6, 117.9}, {20.7, 77.25}, {20.8, 57.35}, {20.9, 59.6}, {21., 55.25}}
```

```
plot_LiF,Kα,2 = With[{d = data_LiF,Kα,2}, ErrorListPlot[
    Table[{v, ErrorBar[Sqrt[v[[2]] / QuantityMagnitude[t_tor,b]]]}, {v, d}],
    Joined → True, InterpolationOrder → 2, PlotStyle → LightGray,
    MeshStyle → ColorData[1, 1], Mesh → Full, PlotRange → All, ImageSize → Full,
    AxesOrigin → {d[[1, 1]] - .01, 0}, AxesLabel → {"β [°]", "A [Bq]"},
    PlotLabel → Style["Kα-Linie 2. Ordnung (via LiF-Kristall)", Bold]]];
```

```
fit_LiF,Kα,2 = With[{d = Select[data_LiF,Kα,2, 20.2 ≤ #[[1]] ≤ 20.6 &]},
  NonlinearModelFit[d, Max[d[[All, 2]]] Exp[-1/2 (x - μ)^2/σ^2],
   {{μ, Mean[d[[All, 1]]]}, {σ, StandardDeviation[d[[All, 1]]]}}, {x}
   , VarianceEstimatorFunction → (1 &),
   Weights → Sqrt[QuantityMagnitude[t_tor,b]]^2/Sqrt[d[[All, 2]]]^2]]
```

```
FittedModel[218.55 e^(-14.3465 ≪1≫^2)]
```

```
Show[plot_LiF,Kα,2, Plot[fit_LiF,Kα,2[x], {x, 19.8, 21}]]
```



```
fit_{LiF,K_α,2}["ParameterTable"]
```

	Estimate	Standard Error	t-Statistic	P-Value
μ	20.3999	0.00211093	9663.95	2.44347×10^{-12}
σ	0.186686	0.00220869	84.5233	3.65025×10^{-6}

```
{λ_{K_α,2}, Δλ_{K_α,2,fit}} =
 λ_LiF[#1, #2, 2] & @@ (fit_{LiF,K_α,2}["ParameterTableEntries"][[1, 1 ;; 2]] °)
```

```
{70.2022 pm, 0.00695474 pm}
```

FWHM (mit Fit-Fehler):

```
{Δλ_{K_α,2}, Δλ_{K_α,2,Δfit}} =
 2 √(2 Log[2]) λ_LiF[#1, #2, 2] & @@ (fit_{LiF,K_α,2}["ParameterTableEntries"][[2, 1 ;; 2]] °)
```

```
{1.54528 pm, 0.0182822 pm}
```

K_β-Linie 2. Ordnung

```
data_{LiF,K_β,2} = Import["/Users/jannis/Dropbox/uniself/AP2/2.2/255
    Röntgenspektrometer/1b/2beta/data.dat"]
```

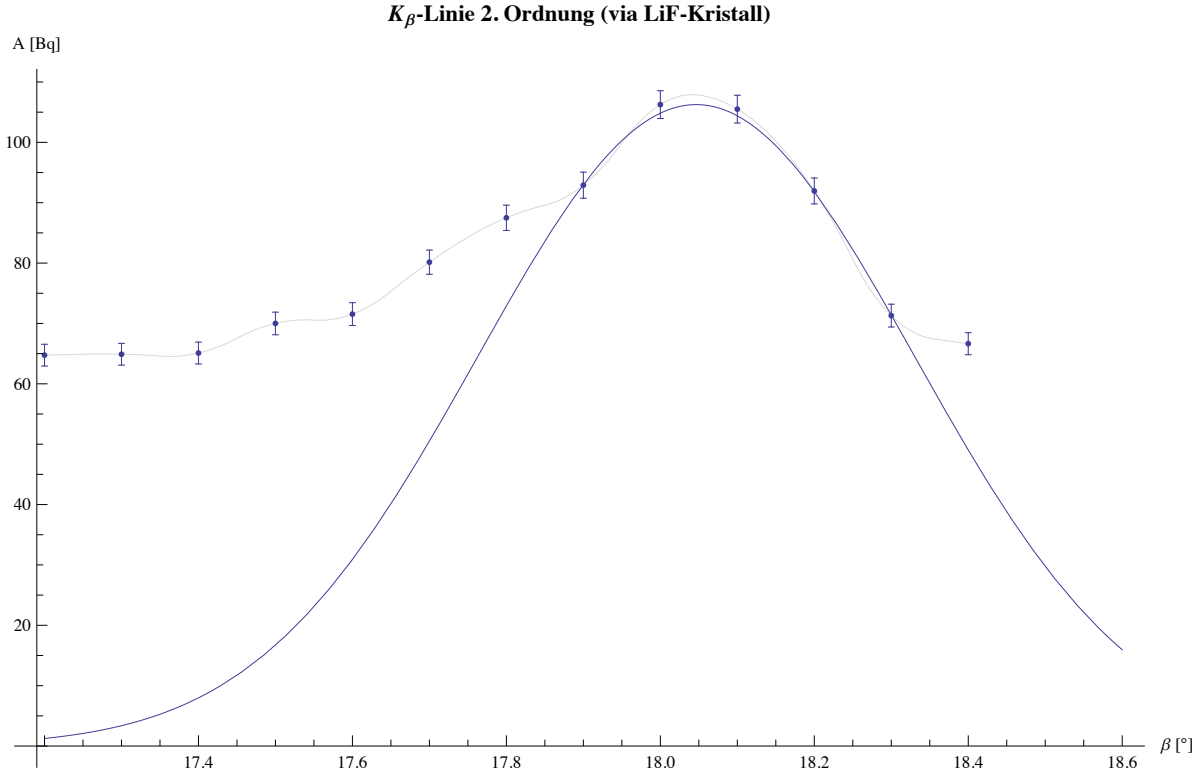
```
{{17.2, 64.75}, {17.3, 64.9}, {17.4, 65.1}, {17.5, 70.},
 {17.6, 71.55}, {17.7, 80.15}, {17.8, 87.5}, {17.9, 92.9}, {18., 106.25},
 {18.1, 105.5}, {18.2, 91.95}, {18.3, 71.3}, {18.4, 66.65}}
```

```
plot_{LiF,K_β,2} = With[{d = data_{LiF,K_β,2}}, ErrorListPlot[
    Table[{v, ErrorBar[Sqrt[v[[2]] / QuantityMagnitude[t_{tor,b}]]]}, {v, d}],
    Joined → True, InterpolationOrder → 2, PlotStyle → LightGray,
    MeshStyle → ColorData[1, 1], Mesh → Full, PlotRange → All, ImageSize → Full,
    AxesOrigin → {d[[1, 1]] - .01, 0}, AxesLabel → {"β [°]", "A [Bq]"},
    PlotLabel → Style["K_β-Linie 2. Ordnung (via LiF-Kristall)", Bold]]];
```

```
fit_{LiF,K_β,2} = With[{d = Select[data_{LiF,K_β,2}, 17.9 ≤ #[[1]] ≤ 18.3 &]},
  NonlinearModelFit[d, Max[d[[All, 2]]] Exp[-1/2 (x - μ)^2/σ^2],
   {{μ, Mean[d[[All, 1]]]}, {σ, StandardDeviation[d[[All, 1]]]}}, {x},
   VarianceEstimatorFunction → (1 &), Weights → Sqrt[QuantityMagnitude[t_{tor,b}]]^2/Sqrt[d[[All, 2]]]^2]]
```

```
FittedModel[106.25 e^(-6.19199 ≪1≫^2)]
```

```
Show[plot_{LiF,Kβ,2}, Plot[fit_{LiF,Kβ,2}[x], {x, 17.2, 18.6}]]
```




```
fit_{LiF,Kβ,2}["ParameterTable"]
```

	Estimate	Standard Error	t-Statistic	P-Value
μ	18.0466	0.00767167	2352.37	1.69415×10^{-10}
σ	0.284165	0.0106304	26.7313	0.000114875

```
{λ_{Kβ,2}, Δλ_{Kβ,2,fit}} =
 λ_{LiF}[#1, #2, 2] & @@ (fit_{LiF,Kβ,2}["ParameterTableEntries"][[1, 1 ;; 2]] °)
```

{62.3919 pm, 0.02564 pm}

FWHM (mit Fit-Fehler):

```
{Δλ_{Kβ,2}, Δλ_{Kβ,2,Δfit}} =
 2 √(2 Log[2]) λ_{LiF}[#1, #2, 2] & @@ (fit_{LiF,Kβ,2}["ParameterTableEntries"][[2, 1 ;; 2]] °)
```

{2.35214 pm, 0.0879911 pm}

h-Bestimmung durch Extrapolation der Einsatzspannung

```
t_{tor,c} = Quantity[20, "Seconds"]
```

20 s

```
va = {
   {Quantity[20, "Kilovolts"], Quantity[1.85, "Becquerels"]},
   {Quantity[21, "Kilovolts"], Quantity[3.45, "Becquerels"]},
   {Quantity[22, "Kilovolts"], Quantity[6.70, "Becquerels"]},
   {Quantity[23, "Kilovolts"], Quantity[27.40, "Becquerels"]},
   {Quantity[24, "Kilovolts"], Quantity[66.60, "Becquerels"]},
   {Quantity[25, "Kilovolts"], Quantity[101.5, "Becquerels"]},
   {Quantity[26, "Kilovolts"], Quantity[138.0, "Becquerels"]},
   {Quantity[27, "Kilovolts"], Quantity[165.4, "Becquerels"]},
   {Quantity[28, "Kilovolts"], Quantity[203.1, "Becquerels"]},
   {Quantity[29, "Kilovolts"], Quantity[228.7, "Becquerels"]},
   {Quantity[30, "Kilovolts"], Quantity[260.9, "Becquerels"]},
   {Quantity[31, "Kilovolts"], Quantity[294.7, "Becquerels"]},
   {Quantity[32, "Kilovolts"], Quantity[316.4, "Becquerels"]},
   {Quantity[33, "Kilovolts"], Quantity[353.1, "Becquerels"]},
   {Quantity[34, "Kilovolts"], Quantity[378.4, "Becquerels"]},
   {Quantity[35, "Kilovolts"], Quantity[407.8, "Becquerels"]}
  }
```

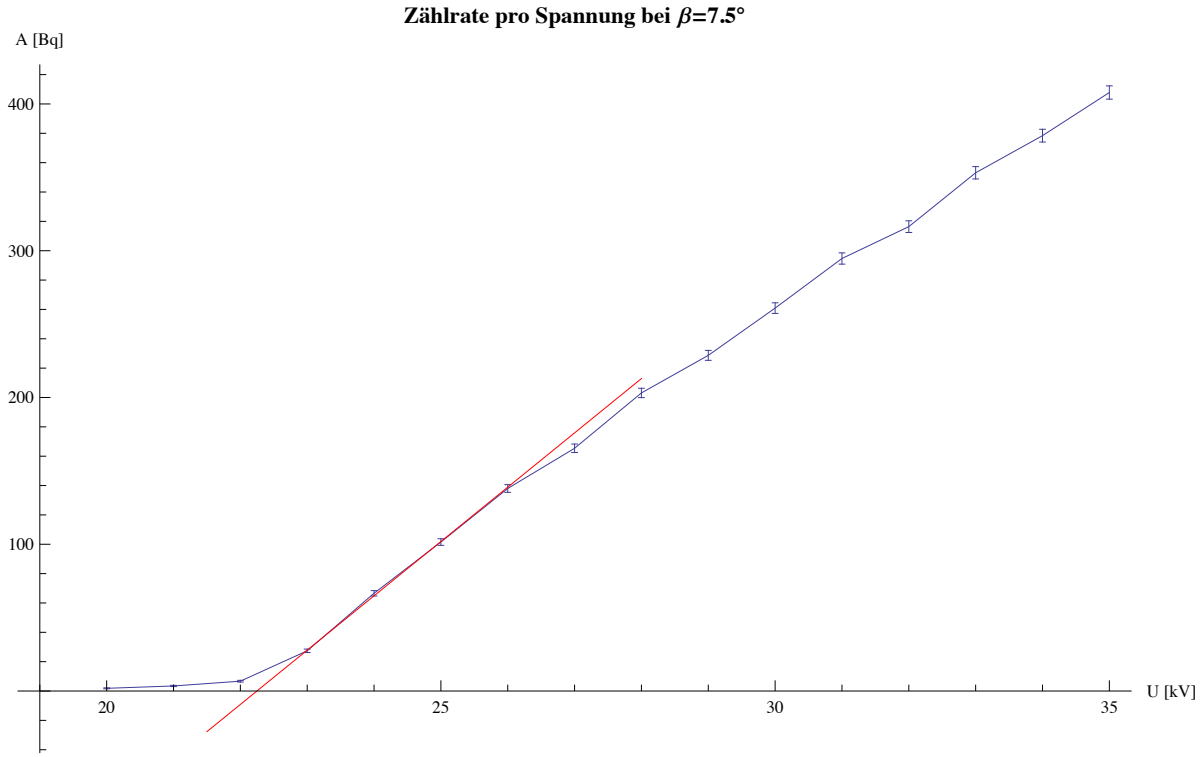
```
{{20 kV, 1.85 Bq}, {21 kV, 3.45 Bq}, {22 kV, 6.7 Bq}, {23 kV, 27.4 Bq},
 {24 kV, 66.6 Bq}, {25 kV, 101.5 Bq}, {26 kV, 138. Bq}, {27 kV, 165.4 Bq},
 {28 kV, 203.1 Bq}, {29 kV, 228.7 Bq}, {30 kV, 260.9 Bq}, {31 kV, 294.7 Bq},
 {32 kV, 316.4 Bq}, {33 kV, 353.1 Bq}, {34 kV, 378.4 Bq}, {35 kV, 407.8 Bq}}
```

```
plot_U = With[{d = va}, ErrorListPlot[
     Table[{QuantityMagnitude /@ v, ErrorBar[
        Sqrt[QuantityMagnitude[v[[2]]] / QuantityMagnitude[t_tor,c]]]}, {v, d}],
     Joined → True, PlotRange → All, ImageSize → Full, AxesOrigin →
      {QuantityMagnitude[d[[1, 1]]] - 1, 0}, AxesLabel → {"U [kV]", "A [Bq]"},
     PlotLabel → Style["Zählrate pro Spannung bei β=7.5°", Bold]]];
```

```
extrapolation_U =
 With[{d = QuantityMagnitude /@ Select[va, 23 ≤ QuantityMagnitude[#[[1]]] < 27 &]},
  LinearModelFit[d, x, x, VarianceEstimatorFunction → (1 &),
   Weights → Sqrt[QuantityMagnitude[t_tor,c]]^2 / Sqrt[d[[All, 2]]]^2]]
```

```
FittedModel[-823.374 + 37.0096 x]
```

```
Show[plot_U, Plot[extrapolation_U[x], {x, 21.5, 28}, PlotStyle → Red]]
```




```
extrapolation_U["ParameterTable"]
```

	Estimate	Standard Error	t-Statistic	P-Value
1	−823.374	19.7099	−41.7746	0.000572536
x	37.0096	0.826576	44.7745	0.00049844

```
{cU, ΔcU} = extrapolation_U["ParameterTableEntries"][[1, 1 ;; 2]]
```

```
{-823.374, 19.7099}
```

```
{mU, ΔmU} = extrapolation_U["ParameterTableEntries"][[2, 1 ;; 2]]
```

```
{37.0096, 0.826576}
```

$$\mathbf{U_{grenz,LiF} = Quantity\left[-\frac{cU}{mU}, "Kilovolts"\right]}$$

```
22.2476 kV
```

$$\mathbf{\Delta U_{grenz,LiF} = Quantity\left[Sqrt\left[\left(\frac{cU\ \Delta mU}{mU^2}\right)^2 + \left(\frac{\Delta cU}{mU}\right)^2\right], "Kilovolts"\right]}$$

```
0.728364 kV
```

```
PlusMinus[NumberForm[U_grenz,LiF, {3, 1}], NumberForm[ΔU_grenz,LiF, 1]]
```

```
22.2 kV ± 0.7 kV
```

```
λ_7.5° = First[λ_LiF[7.5 °, 0]]
```

```
52.576 pm
```

```
UnitConvert[(Quantity["ElementaryCharge"] * {U_grenz,LiF, ΔU_grenz,LiF} * λ_7.5°)/Quantity["SpeedOfLight"], "Joules" * "Seconds"]
```

$$\{6.25115 \times 10^{-34}\ \mathrm{s\,J},\ 2.04656 \times 10^{-35}\ \mathrm{s\,J}\}$$

Elementare Strukturanalyse eines NaCl-Kristalls

Wellenlängen aus Teil I

$$\{\lambda_{K_\alpha}, \Delta\lambda_{K_\alpha}\} = \left\{\frac{\lambda_{K_\alpha,1} + \lambda_{K_\alpha,2}}{2}, \frac{1}{2}\sqrt{\Delta\lambda_{K_\alpha,1}{}^2 + \Delta\lambda_{K_\alpha,2}{}^2}\right\}$$

$$\{69.4877\ \mathrm{pm},\ 1.54126\ \mathrm{pm}\}$$

$$\{\lambda_{K_\beta}, \Delta\lambda_{K_\beta}\} = \left\{\frac{\lambda_{K_\beta,1} + \lambda_{K_\beta,2}}{2}, \frac{1}{2}\sqrt{\Delta\lambda_{K_\beta,1}{}^2 + \Delta\lambda_{K_\beta,2}{}^2}\right\}$$

$$\{61.3769\ \mathrm{pm},\ 2.27729\ \mathrm{pm}\}$$

Spektrum des NaCl-Kristalls

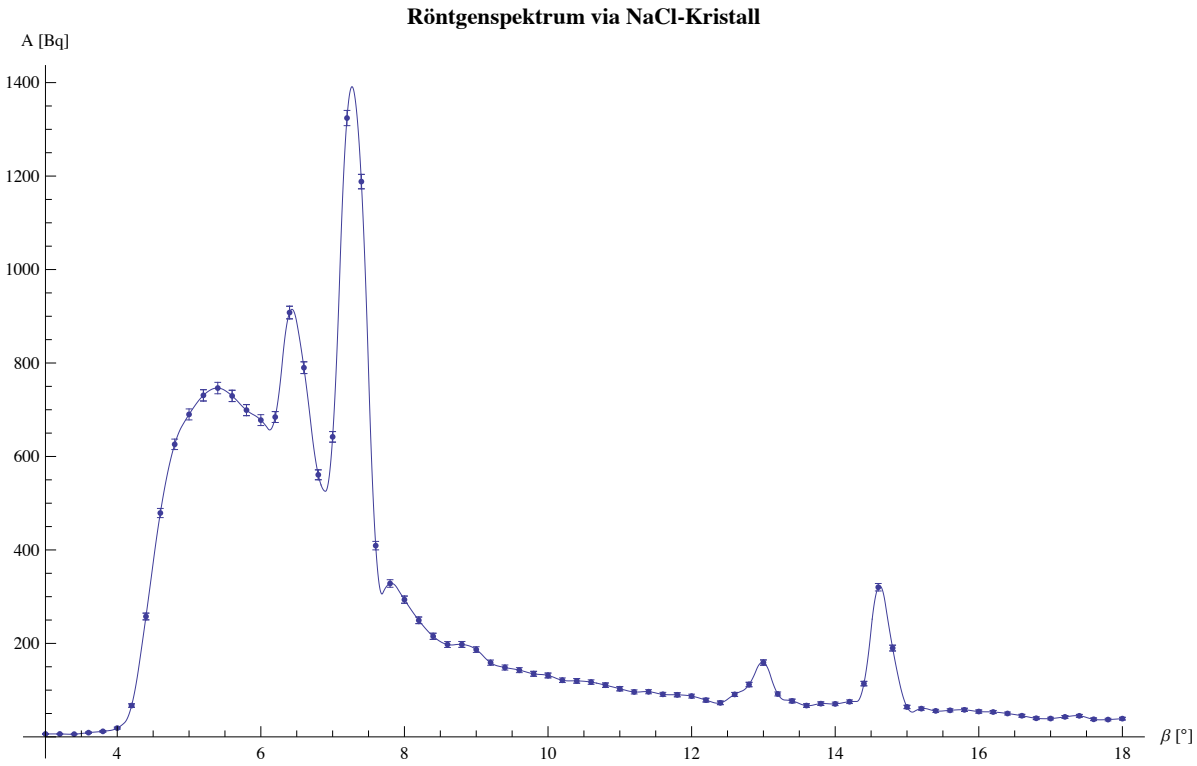
```
spectrum_NaCl = Import[
  "/Users/jannis/Dropbox/uniself/AP2/2.2/255 Röntgenspektrometer/2/data.dat"]
```

```
{{3., 6.2}, {3.2, 6.2}, {3.4, 5.8}, {3.6, 9.}, {3.8, 11.8}, {4., 18.8}, {4.2, 67.},
 {4.4, 257.6}, {4.6, 479.}, {4.8, 626.}, {5., 690.}, {5.2, 730.8}, {5.4, 746.4},
 {5.6, 729.6}, {5.8, 699.2}, {6., 677.8}, {6.2, 684.4}, {6.4, 908.}, {6.6, 790.},
 {6.8, 560.8}, {7., 642.}, {7.2, 1324.2}, {7.4, 1188.2}, {7.6, 409.2},
 {7.8, 328.2}, {8., 293.6}, {8.2, 249.4}, {8.4, 215.2}, {8.6, 197.6},
 {8.8, 197.8}, {9., 186.8}, {9.2, 158.8}, {9.4, 148.4}, {9.6, 142.8},
 {9.8, 135.}, {10., 131.4}, {10.2, 121.2}, {10.4, 119.8}, {10.6, 117.2},
 {10.8, 110.6}, {11., 103.}, {11.2, 96.2}, {11.4, 96.6}, {11.6, 91.},
 {11.8, 90.}, {12., 87.2}, {12.2, 78.6}, {12.4, 72.8}, {12.6, 90.8},
 {12.8, 112.}, {13., 159.}, {13.2, 91.8}, {13.4, 76.8}, {13.6, 67.}, {13.8, 71.},
 {14., 70.6}, {14.2, 75.2}, {14.4, 114.}, {14.6, 320.2}, {14.8, 190.},
 {15., 64.}, {15.2, 60.4}, {15.4, 55.6}, {15.6, 56.8}, {15.8, 57.8},
 {16., 54.2}, {16.2, 53.2}, {16.4, 50.}, {16.6, 45.2}, {16.8, 39.8},
 {17., 39.2}, {17.2, 42.8}, {17.4, 45.}, {17.6, 37.6}, {17.8, 37.}, {18., 39.}}
```

```
t_tor,2 = Quantity[5, "Seconds"]
```

5 s

```
plot_NaCl = With[{d = spectrum_NaCl}, ErrorListPlot[
    Table[{v, ErrorBar[Sqrt[v[[2]] / QuantityMagnitude[t_tor,2]]]}, {v, d}],
    Joined → True, InterpolationOrder → 2, Mesh → Full, PlotRange → All,
    ImageSize → Full, AxesOrigin → {d[[1, 1]], 0}, AxesLabel → {"β [°]", "A [Bq]"},
    PlotLabel → Style["Röntgenspektrum via NaCl-Kristall", Bold]]]
```



$$\text{fit}_{\text{NaCl},K_\alpha,1} = \text{With}\left[\{d = \text{Select}[\text{spectrum}_{\text{NaCl}}, 7 \le \#[[1]] \le 7.8\ \&]\},\ \text{NonlinearModelFit}\left[d, 1386\ \text{Exp}\left[-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}\right], \{\{\mu, \text{Mean}[d[[\text{All}, 1]]]\}, \{\sigma, \text{StandardDeviation}[d[[\text{All}, 1]]]\}\}, \{x\}, \text{VarianceEstimatorFunction} \to (1\ \&), \text{Weights} \to \frac{\text{Sqrt}[\text{QuantityMagnitude}[t_{\text{tor},2}]]^2}{\text{Sqrt}[d[[\text{All}, 2]]]^2}\right]\right]$$

$$\text{FittedModel}\left[1386\, e^{-9.17932 \ll 1 \gg^2}\right]$$

$$\text{fit}_{\text{NaCl},K_\beta,1} = \text{With}\left[\{d = \text{Select}[\text{spectrum}_{\text{NaCl}}, 6.2 \le \#[[1]] \le 6.8\ \&]\},\ \text{NonlinearModelFit}\left[d, \text{Max}[d[[\text{All}, 2]]]\ \text{Exp}\left[-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}\right], \{\{\mu, \text{Mean}[d[[\text{All}, 1]]]\}, \{\sigma, \text{StandardDeviation}[d[[\text{All}, 1]]]\}\}, \{x\}, \text{VarianceEstimatorFunction} \to (1\ \&), \text{Weights} \to \frac{\text{Sqrt}[\text{QuantityMagnitude}[t_{\text{tor},2}]]^2}{\text{Sqrt}[d[[\text{All}, 2]]]^2}\right]\right]$$

$$\text{FittedModel}\left[908.\, e^{-4.19578 \ll 1 \gg^2}\right]$$

```
fit_{NaCl,K_α,2} = With[{d = Select[spectrum_{NaCl}, 14.4 ≤ #[[1]] ≤ 15 &]},
  NonlinearModelFit[d, Max[d[[All, 2]]] Exp[-1/2 (x - μ)^2/σ^2],
   {{μ, Mean[d[[All, 1]]]}, {σ, StandardDeviation[d[[All, 1]]]}}, {x},
   VarianceEstimatorFunction → (1 &), Weights → Sqrt[QuantityMagnitude[t_{tor,2}]]^2/Sqrt[d[[All, 2]]]^2]]
```

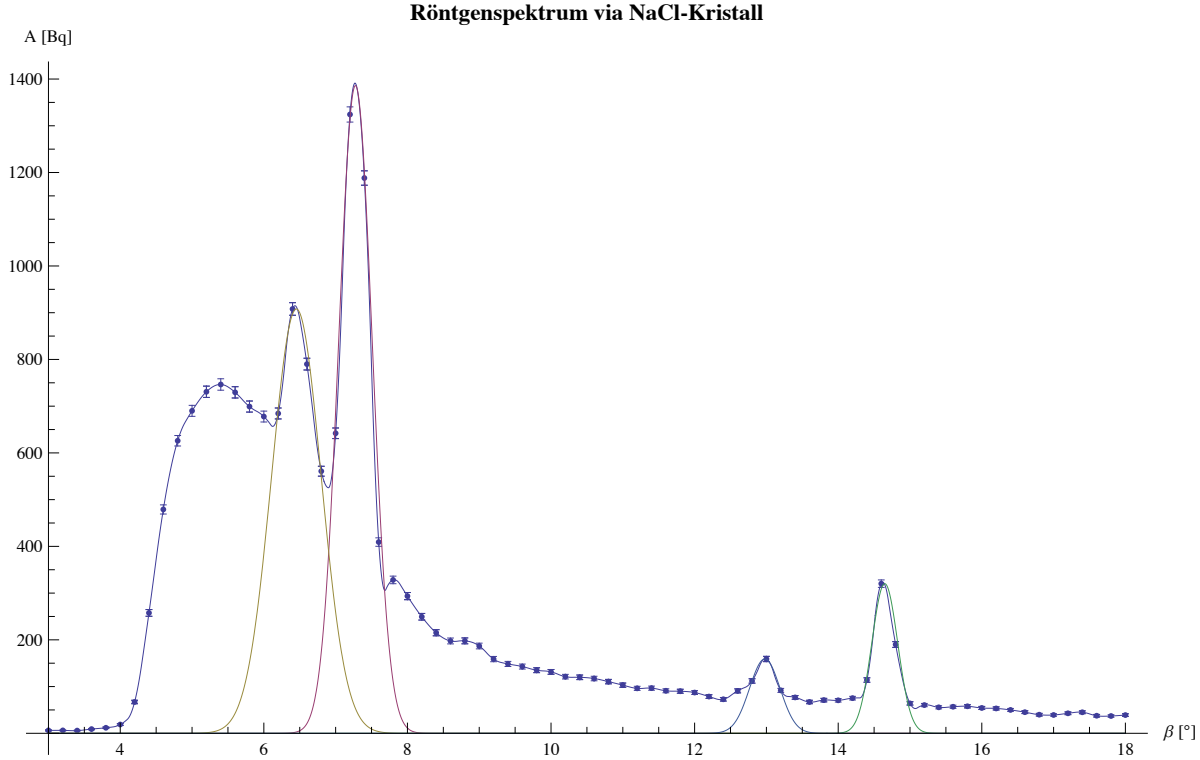
FittedModel[$320.2\, e^{-15.4695 \ll 1 \gg^2}$]

```
fit_{NaCl,K_β,2} = With[{d = Select[spectrum_{NaCl}, 12.8 ≤ #[[1]] ≤ 13.2 &]},
  NonlinearModelFit[d, Max[d[[All, 2]]] Exp[-1/2 (x - μ)^2/σ^2],
   {{μ, Mean[d[[All, 1]]]}, {σ, StandardDeviation[d[[All, 1]]]}}, {x},
   VarianceEstimatorFunction → (1 &), Weights → Sqrt[QuantityMagnitude[t_{tor,2}]]^2/Sqrt[d[[All, 2]]]^2]]
```

FittedModel[$159.\, e^{-11.1086 \ll 1 \gg^2}$]

```
Show[plot_{NaCl},
 Plot[fit_{NaCl,K_α,1}[x], {x, 3, 18}, PlotRange → Full, PlotStyle → ColorData[1, 2]],
 Plot[fit_{NaCl,K_β,1}[x], {x, 3, 18}, PlotRange → Full, PlotStyle → ColorData[1, 3]],
 Plot[fit_{NaCl,K_α,2}[x], {x, 3, 18}, PlotRange → Full, PlotStyle → ColorData[1, 4]],
 Plot[fit_{NaCl,K_β,2}[x], {x, 3, 18}, PlotRange → Full, PlotStyle → ColorData[1, 5]]]
```

Röntgenspektrum via NaCl-Kristall



```
fit_{NaCl,K_α,1}["ParameterTable"]
```

	Estimate	Standard Error	t-Statistic	P-Value
μ	7.27589	0.00198986	3656.49	4.51105×10^{-11}
σ	0.233389	0.00153871	151.679	6.31874×10^{-7}

```
{β_{NaCl,K_α,1}, Δβ_{NaCl,K_α,1}} =
  ({1, √(2 Log[2])} fit_{NaCl,K_α,1}["ParameterTableEntries"][[1 ;; 2, 1]])
```

```
{7.27589, 0.274794}
```

```
fit_{NaCl,K_β,1}["ParameterTable"]
```

	Estimate	Standard Error	t-Statistic	P-Value
μ	6.45325	0.00498637	1294.18	5.97053×10^{-7}
σ	0.345206	0.00592204	58.2918	0.000294166

```
{β_{NaCl,K_β,1}, Δβ_{NaCl,K_β,1}} =
  ({1, √(2 Log[2])} fit_{NaCl,K_β,1}["ParameterTableEntries"][[1 ;; 2, 1]])
```

```
{6.45325, 0.406449}
```

```
fit_{NaCl,K_α,2}["ParameterTable"]
```

	Estimate	Standard Error	t-Statistic	P-Value
μ	14.6486	0.00329677	4443.33	5.06504×10^{-8}
σ	0.179782	0.00239504	75.0643	0.000177426

```
{β_{NaCl,K_α,2}, Δβ_{NaCl,K_α,2}} =
  ({1, √(2 Log[2])} fit_{NaCl,K_α,2}["ParameterTableEntries"][[1 ;; 2, 1]])
```

```
{14.6486, 0.211677}
```

```
fit_{NaCl,K_β,2}["ParameterTable"]
```

	Estimate	Standard Error	t-Statistic	P-Value
μ	12.9777	0.0072464	1790.92	0.000355471
σ	0.212156	0.00757296	28.0149	0.0227147

```
{β_{NaCl,K_β,2}, Δβ_{NaCl,K_β,2}} =
  ({1, √(2 Log[2])} fit_{NaCl,K_β,2}["ParameterTableEntries"][[1 ;; 2, 1]])
```

```
{12.9777, 0.249794}
```

```
d[λ_, Δλ_, β_, Δβ_, n_: 1] := {n λ / (2 Sin[β]), √((n Δλ / (2 Sin[β]))^2 + (n λ Cos[β] Δβ / (2 Sin[β]^2))^2)}
```

```
ds = Transpose[{d[λ_{K_α,1}, Δλ_{K_α,1}, β_{NaCl,K_α,1}°, Δβ_{NaCl,K_α,1}°],
    d[λ_{K_β,1}, Δλ_{K_β,1}, β_{NaCl,K_β,1}°, Δβ_{NaCl,K_β,1}°],
    d[λ_{K_α,2}, Δλ_{K_α,2}, β_{NaCl,K_α,2}°, Δβ_{NaCl,K_α,2}°, 2],
    d[λ_{K_β,2}, Δλ_{K_β,2}, β_{NaCl,K_β,2}°, Δβ_{NaCl,K_β,2}°, 2]}]
```

```
{{271.515 pm, 268.532 pm, 277.599 pm, 277.825 pm},
 {14.6598 pm, 24.1803 pm, 7.26169 pm, 11.7186 pm}}
```

$$\{d_{NaCl}, \Delta d_{NaCl}\} = \left\{\text{Mean}[ds[[1]]], \frac{1}{4}\sqrt{\text{Total}\left[ds[[2]]^2\right]}\right\}$$

$\{273.868\ \text{pm}, 7.8647\ \text{pm}\}$

$$\{a_{NaCl}, \Delta a_{NaCl}\} = 2\ \{d_{NaCl}, \Delta d_{NaCl}\}$$

$\{547.735\ \text{pm}, 15.7294\ \text{pm}\}$

$$\rho_{NaCl} = \text{Quantity}\left[2.164, \frac{\text{"Grams"}}{\text{"Centimeters"}^3}\right]$$

$2.164\ \text{g}/\text{cm}^3$

$$M_{NaCl} = \text{Quantity}[58.44, \text{"Grams"}]$$

$58.44\ \text{g}$

$$\{\text{Avogadro}, \Delta\text{Avogadro}\} = \frac{1}{2}\ \frac{M_{NaCl}}{\rho_{NaCl}\ d_{NaCl}^3}\left\{1, 3\ \frac{\Delta d_{NaCl}}{d_{NaCl}}\right\}$$

$\left\{6.57356 \times 10^{23}, 8.59906 \times 10^{22}\right\}$