251 Statistik des radioaktiven Zerfalls

In[115]:=
```
Needs["ErrorBarPlots`"]
```

Untersuchung des Plateau-Anstiegs

In[116]:= U_0 = Quantity[700, "Volts"]

Out[116]= 700 V

In[117]:= $n_{1,700} = 8869$

Out[117]= 8869

In[118]:= $n_{1,800} = 9092$

Out[118]= 9092

In[119]:= $n_{3,700} = 26\,601$

Out[119]= 26 601

In[120]:= $n_{3,800} = 27\,320$

Out[120]= 27 320

In[121]:= $\{\delta_1, \Delta\delta_1\} = \{n_{1,800} - n_{1,700}, \text{Sqrt}[n_{1,800} + n_{1,700}]\}$ // N

Out[121]= {223., 134.019}

In[122]:= $\{\delta_3, \Delta\delta_3\} = \{n_{3,800} - n_{3,700}, \text{Sqrt}[n_{3,800} + n_{3,700}]\}$ // N

Out[122]= {719., 232.209}

Prozentualer Anstieg:

In[123]:= $\frac{\{\delta_1, \Delta\delta_1\}}{n_{1,700}}$

Out[123]= {0.0251438, 0.0151109}

In[124]:= $\frac{\{\delta_3, \Delta\delta_3\}}{n_{3,700}}$

Out[124]= {0.0270291, 0.00872933}

Variation bei ca. 68% Vertrauen (1 σ):

In[125]:= $\frac{\delta_1 + \{-\Delta\delta_1, +\Delta\delta_1\}}{n_{1,700}}$

Out[125]= {0.0100328, 0.0402547}

In[126]:= $\frac{\delta_3 + \{-\Delta\delta_3, +\Delta\delta_3\}}{n_{3,700}}$

Out[126]= {0.0182997, 0.0357584}

Variation bei ca. 95% Vertrauen (2 σ):

In[127]:= $\frac{\delta_1 + \{-2\,\Delta\delta_1, +2\,\Delta\delta_1\}}{n_{1,700}}$

Out[127]= {-0.00507806, 0.0553656}

In[128]:= $\frac{\delta_3 + \{-2\,\Delta\delta_3, +2\,\Delta\delta_3\}}{n_{3,700}}$

Out[128]= {0.0095704, 0.0444877}

Genauigkeit des prozentualen Anstiegs:

In[129]:= $\frac{\Delta\delta_3}{n_{3,700}}$

Out[129]= 0.00872933

In[168]:= $\frac{\Delta\delta_3}{n_{3,700}}$ * Sqrt[3 * 60]

Out[168]= 0.117116

In[167]:= Solve[$\frac{\Delta\delta_3}{n_{3,700}}$ * Sqrt[3 * 60] / Sqrt[t] == 1 / 300, t]

Out[167]= {{t → 1234.46}}

Auswertung der Daten mit hoher mittlerer Ereigniszahl

In[130]:= $data_{large}$ = Import["/Users/jannis/Dropbox/uniself/AP2/2.2/251 Statistik/JJ4.dat"]

Out[130]= {{43, 1}, {44, 2}, {45, 1}, {46, 1}, {47, 0}, {48, 1}, {49, 1}, {50, 4}, {51, 5}, {52, 8}, {53, 13}, {54, 13}, {55, 7}, {56, 15}, {57, 24}, {58, 23}, {59, 33}, {60, 48}, {61, 40}, {62, 44}, {63, 60}, {64, 61}, {65, 72}, {66, 57}, {67, 76}, {68, 86}, {69, 92}, {70, 85}, {71, 84}, {72, 91}, {73, 81}, {74, 75}, {75, 84}, {76, 69}, {77, 62}, {78, 62}, {79, 57}, {80, 58}, {81, 56}, {82, 52}, {83, 45}, {84, 43}, {85, 39}, {86, 28}, {87, 21}, {88, 18}, {89, 23}, {90, 18}, {91, 18}, {92, 11}, {93, 8}, {94, 8}, {95, 8}, {96, 6}, {97, 3}, {98, 1}, {99, 1}, {100, 3}, {101, 1}, {102, 3}, {103, 0}, {104, 1}, {105, 0}, {106, 0}, {107, 0}, {108, 0}, {109, 1}}

In[131]:= n_{large} = Total[$data_{large}$[[All, 2]]]

Out[131]= 2012

In[132]:= $total_{large}$ = Total[$data_{large}$[[All, 1]] * $data_{large}$[[All, 2]]]

Out[132]= 145 981

In[133]:= $mean_{large}$ = N[$total_{large}$ / n_{large}]

Out[133]= 72.5552

In[240]:=
```
stdev_large = N@Sqrt[Total[data_large[[All, 1]]^2 * data_large[[All, 2]] / n_large] -
    (Total[data_large[[All, 1]] * data_large[[All, 2]] / n_large])^2]
```
Out[240]= 9.51772

In[134]:=
```
t_tor,large = Quantity[0.5, "Seconds"]
```
Out[134]= 0.5 s

In[135]:=
```
ϵ_large = 0;
```

In[136]:=
```
errors_large = Map[If[# == 0, ϵ_large, #] &, Sqrt[data_large[[All, 2]]]] // N
```
Out[136]=
```
{1., 1.41421, 1., 1., 0., 1., 1., 2., 2.23607, 2.82843, 3.60555, 3.60555,
 2.64575, 3.87298, 4.89898, 4.79583, 5.74456, 6.9282, 6.32456, 6.63325,
 7.74597, 7.81025, 8.48528, 7.54983, 8.7178, 9.27362, 9.59166, 9.21954,
 9.16515, 9.53939, 9., 8.66025, 9.16515, 8.30662, 7.87401, 7.87401, 7.54983,
 7.61577, 7.48331, 7.2111, 6.7082, 6.55744, 6.245, 5.2915, 4.58258,
 4.24264, 4.79583, 4.24264, 4.24264, 3.31662, 2.82843, 2.82843, 2.82843,
 2.44949, 1.73205, 1., 1., 1.73205, 1., 1.73205, 0., 1., 0., 0., 0., 0., 1.}
```

In[177]:=
```
plot_data,large = ErrorListPlot[Transpose[{data_large, ErrorBar /@ errors_large}],
   PlotRange → {{40, 110}, {0, 110}},
   AxesLabel → {"Zerfälle pro 500 ms", "Anzahl Messungen"}, PlotLabel →
    "Histogramm für 2012 Messungen bei t_tor = 0.5 s", ImageSize → Full];
```

Für den Fit wählen wir nur Werte ≥ 10.

In[138]:=
```
d_large = Select[data_large, Last[#] ≥ 10 &];
```

In[139]:=
```
δ_large = Select[errors_large, # ≥ Sqrt[10] &];
```

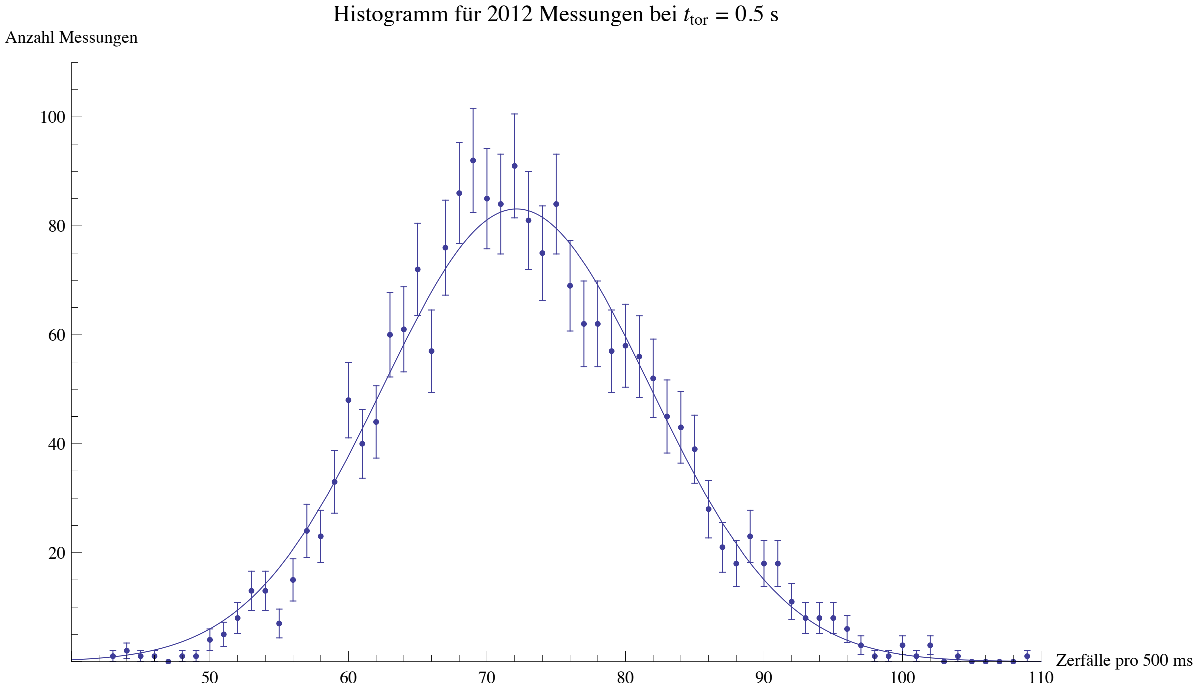
In[241]:=
```
f_gauss,large = NonlinearModelFit[d_large,
  n_large / Sqrt[2 π σ^2] Exp[-(x - μ)^2 / (2 σ^2)], {{σ, stdev_large}, {μ, mean_large}}, {x},
  VarianceEstimatorFunction → 1 &; Weights → 1 / δ_large^2]
```
Out[241]= FittedModel[$83.0994\, e^{-\ll 21 \gg (\ll 1 \gg)^2}$]

In[242]:=
```
f_gauss,large["ParameterTable"]
```
Out[242]=

	Estimate	Standard Error	t-Statistic	P-Value
σ	9.65918	0.197782	48.8374	3.32868×10^{-35}
μ	72.138	0.21124	341.497	2.44813×10^{-66}

In[243]:=
```
Show[plot_{data,large}, Plot[f_{gauss,large}[x], {x, 40, 110}]]
```

Out[243]=



χ^2:

In[244]:=
$$\left\{\frac{1}{\text{Length}[d_{large}] - 3}, 1\right\}\ \text{Total}\left[\text{Table}\left[\frac{\left(d_{large}[[i, 2]] - f_{gauss,large}[d_{large}[[i, 1]]]\right)^2}{\delta_{large}[[i]]^2}, \{i, 1, \text{Length}[d_{large}]\}\right]\right]$$

Out[244]= {0.735904, 26.4925}

In[251]:=
```
1 - CDF[ChiSquareDistribution[Length[d_large] - 3]][26.492538130484558`]
```

Out[251]= 0.876415

Poisson-Fit

In[144]:=
$$f_{poisson,large} = \text{NonlinearModelFit}\left[d_{large}, n_{large}\ \text{Exp}[-\mu]\ \frac{\mu^x}{\text{Gamma}[x+1]}, \{\{\mu, mean_{large}\}\}, \{x\}, \text{VarianceEstimatorFunction} \to 1\ \&; \text{Weights} \to \frac{1}{\delta_{large}{}^2}\right]$$

Out[144]= FittedModel$\left[\frac{8.09802\times10^{-29} \ll 18 \gg^x}{\text{Gamma}[1+x]}\right]$

In[145]:=
```
f_poisson,large["ParameterTable"]
```

Out[145]=

	Estimate	Standard Error	t-Statistic	P-Value
μ	72.2902	0.255561	282.869	9.14199×10^{-65}

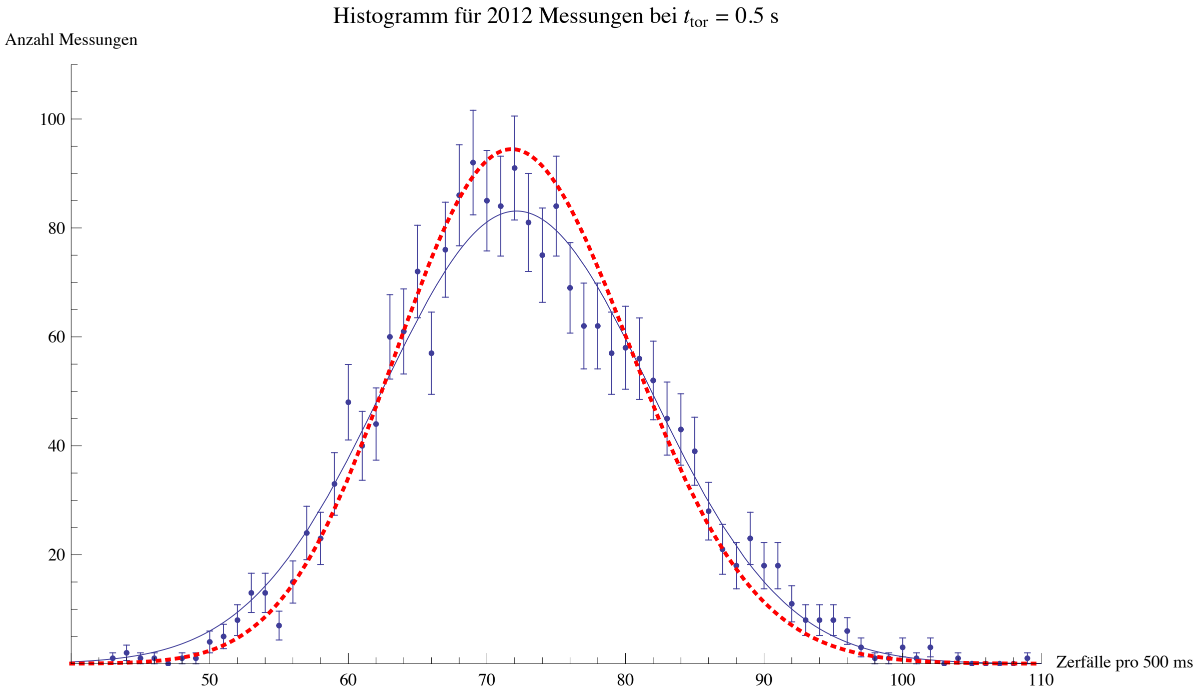
$\sigma = \sqrt{\mu} \pm \frac{\Delta\mu}{2\sqrt{\mu}}$:

In[179]:=
```
{Sqrt[f_{poisson,large}["ParameterTableEntries"][[1, 1]]],
 f_{poisson,large}["ParameterTableEntries"][[1, 2]] /
  (2 Sqrt[f_{poisson,large}["ParameterTableEntries"][[1, 1]]])}
```

Out[179]= {8.50237, 0.0150288}

In[178]:=
```
Show[plot_{data,large}, Plot[f_{gauss,large}[x], {x, 40, 110}],
 Plot[f_{poisson,large}[x], {x, 40, 110}, PlotStyle → {Thick, Dotted, Red}]]
```

Out[178]=



χ^2:

In[220]:=
```
{1/(Length[d_large] - 2), 1}
 Total[Table[(d_large[[i, 2]] - f_{poisson,large}[d_large[[i, 1]]])^2 / δ_large[[i]]^2, {i, 1, Length[d_large]}]]
```

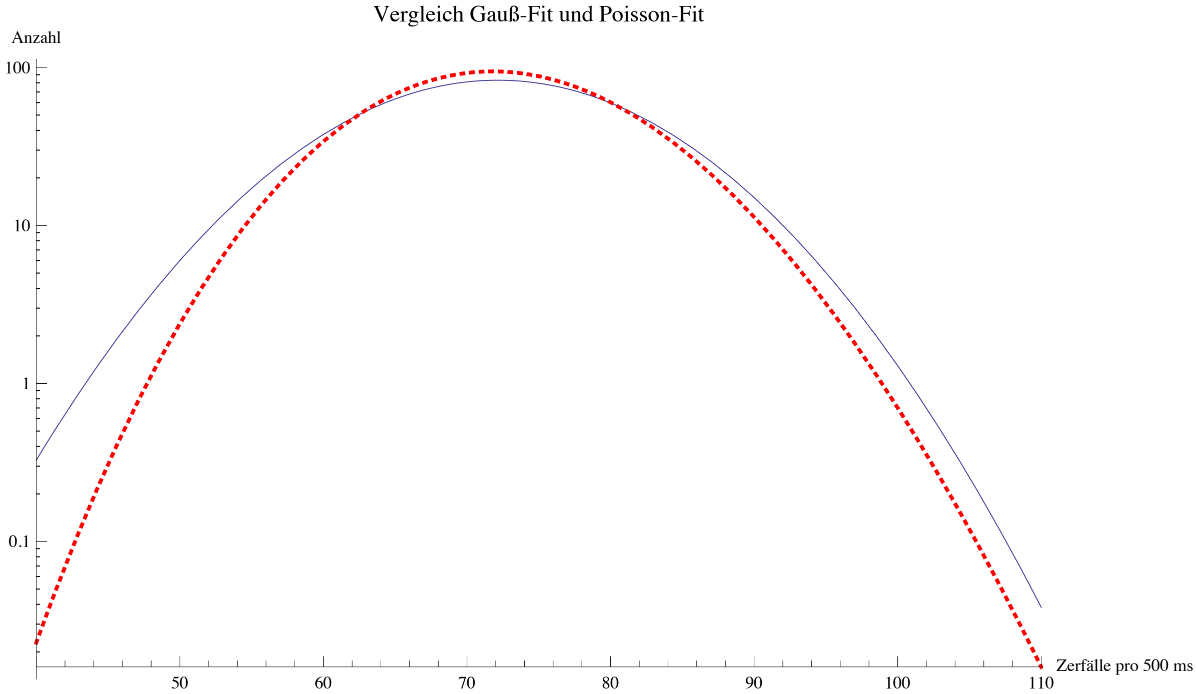
Out[220]= {1.41966, 52.5274}

In[252]:=
```
1 - CDF[ChiSquareDistribution[Length[d_large] - 2]][52.52743606738058`]
```

Out[252]= 0.0468871

In[212]:=
```
Show[LogPlot[{f_gauss,large[x], f_poisson,large[x]},
  {x, 40, 110}, PlotStyle → {Automatic, {Thick, Dotted, Red}},
  ImageSize → Full, PlotLabel → "Vergleich Gauß-Fit und Poisson-Fit",
  AxesLabel → {"Zerfälle pro 500 ms", "Anzahl"},
  PlotRange → {{40, 110}, Automatic}] (*,
 ListLogPlot[Map[(If[#[[2]]==0,{#[[1]],0.017},#]&),data_large],
  Joined→True,InterpolationOrder→0]*)]
```

Out[212]=



Auswertung der Daten mit kleiner mittlerer Ereigniszahl

In[148]:=
```
data_small = Import["/Users/jannis/Dropbox/uniself/AP2/2.2/251 Statistik/JJ5.dat"]
```

Out[148]= {{0, 44}, {1, 252}, {2, 506}, {3, 787}, {4, 890}, {5, 911}, {6, 660}, {7, 430}, {8, 264}, {9, 138}, {10, 72}, {11, 25}, {12, 18}, {13, 4}, {14, 1}, {15, 1}}

In[149]:=
```
n_small = Total[data_small[[All, 2]]]
```

Out[149]= 5003

In[150]:=
```
total_small = Total[data_small[[All, 1]] * data_small[[All, 2]]]
```

Out[150]= 23 356

In[151]:=
```
mean_small = N[total_small / n_small]
```

Out[151]= 4.6684

In[245]:=
$$\text{stdev}_{\text{small}} = \text{N@Sqrt}\left[\text{Total}\left[\text{data}_{\text{small}}[[\text{All}, 1]]^2 * \frac{\text{data}_{\text{small}}[[\text{All}, 2]]}{n_{\text{small}}}\right] - \left(\text{Total}\left[\text{data}_{\text{small}}[[\text{All}, 1]] * \frac{\text{data}_{\text{small}}[[\text{All}, 2]]}{n_{\text{small}}}\right]\right)^2\right]$$

Out[245]= 2.19738

In[152]:=
```
t_tor,small = Quantity[0.1, "Seconds"]
```
Out[152]= 0.1 s

In[153]:=
```
ϵ_small = 0;
```

In[154]:=
```
errors_small = Map[If[# == 0, ϵ_small, #] &, Sqrt[data_small[[All, 2]]]] // N
```
Out[154]=
```
{6.63325, 15.8745, 22.4944, 28.0535, 29.8329, 30.1828,
 25.6905, 20.7364, 16.2481, 11.7473, 8.48528, 5., 4.24264, 2., 1., 1.}
```

In[259]:=
```
plot_data,small = ErrorListPlot[Transpose[{data_small, ErrorBar /@ errors_small}],
   PlotRange → {{0, 15.1}, {0, 1000}},
   AxesLabel → {"Zerfälle pro 100 ms", "Anzahl Messungen"}, PlotLabel →
    "Histogramm für 5003 Messungen bei t_tor = 0.1 s", ImageSize → Full];
```

Für den Fit wählen wir nur Werte ≥ 10.

In[156]:=
```
d_small = Select[data_small, Last[#] ≥ 10 &];
```

In[157]:=
```
δ_small = Select[errors_small, # ≥ Sqrt[10] &];
```

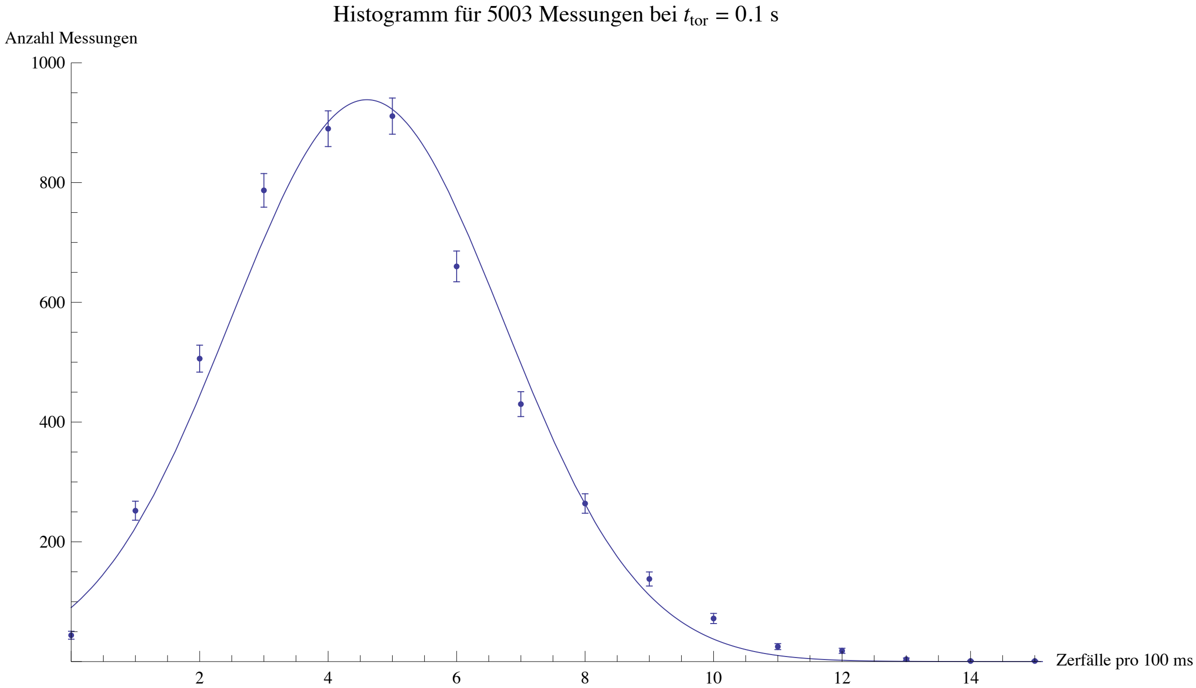
In[246]:=
```
f_gauss,small = NonlinearModelFit[d_small,
  n_small/Sqrt[2 π σ^2] Exp[-(x - μ)^2/(2 σ^2)], {{σ, stdev_small}, {μ, mean_small}}, {x},
  VarianceEstimatorFunction → 1 &; Weights → 1/δ_small^2]
```
Out[246]= FittedModel[$938.408\, e^{-0.110528 \ll 1\gg \ll 1\gg}$]

In[247]:=
```
f_gauss,small["ParameterTable"]
```
Out[247]=

	Estimate	Standard Error	t-Statistic	P-Value
σ	2.12691	0.0851839	24.9684	4.88847×10^{-11}
μ	4.60332	0.108889	42.2753	1.57967×10^{-13}

In[260]:=
```
Show[plot_{data,small}, Plot[f_{gauss,small}[x], {x, 0, 16}]]
```

Out[260]=



χ^2:

In[249]:=
$$\left\{\frac{1}{\text{Length}[d_{small}] - 3}, 1\right\} \text{Total}\left[\text{Table}\left[\frac{(d_{small}[[i, 2]] - f_{gauss,small}[d_{small}[[i, 1]]])^2}{\delta_{small}[[i]]^2}, \{i, 1, \text{Length}[d_{small}]\}\right]\right]$$

Out[249]= {13.7102, 137.102}

In[253]:=
```
1 - CDF[ChiSquareDistribution[Length[d_small] - 3]][137.10215392867764`]
```

Out[253]= 0.

Poisson-Fit

In[162]:=
$$f_{poisson,small} = \text{NonlinearModelFit}\left[d_{small}, n_{small}\,\text{Exp}[-\mu]\,\frac{\mu^x}{\text{Gamma}[x+1]}, \{\{\mu, mean_{small}\}\}, \{x\}, \text{VarianceEstimatorFunction} \to 1\,\&; \text{Weights} \to \frac{1}{\delta_{small}{}^2}\right]$$

Out[162]= FittedModel$\left[\frac{47.1499 \ll 18 \gg^x}{\text{Gamma}[1+x]}\right]$

In[163]:=
```
f_poisson,small["ParameterTable"]
```

Out[163]=

	Estimate	Standard Error	t-Statistic	P-Value
μ	4.66446	0.0336701	138.534	1.3434×10^{-20}

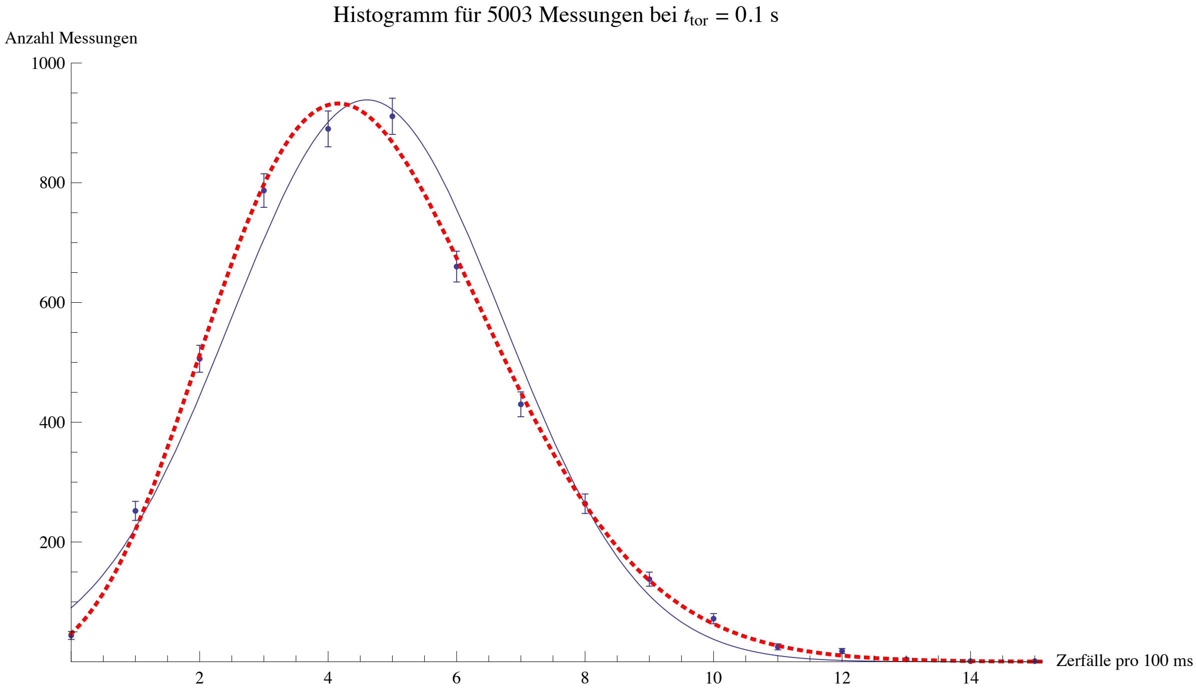
$\sigma = \sqrt{\mu} \pm \frac{\Delta\mu}{2\sqrt{\mu}}$:

In[217]:=
```
{Sqrt[f_poisson,small["ParameterTableEntries"][[1, 1]]],
 f_poisson,small["ParameterTableEntries"][[1, 2]] /
  (2 Sqrt[f_poisson,small["ParameterTableEntries"][[1, 1]]])}
```

Out[217]= {2.15974, 0.00779496}

In[261]:=
```
Show[plot_data,small, Plot[f_gauss,small[x], {x, 0, 16}],
 Plot[f_poisson,small[x], {x, 0, 16}, PlotStyle → {Thick, Dotted, Red}]]
```

Out[261]=



χ^2:

In[257]:=
$$\left\{\frac{1}{\text{Length}[d_{small}] - 2}, 1\right\}$$
$$\text{Total}\left[\text{Table}\left[\frac{(d_{small}[[i, 2]] - f_{poisson,small}[d_{small}[[i, 1]]])^2}{\delta_{small}[[i]]^2}, \{i, 1, \text{Length}[d_{small}]\}\right]\right]$$

Out[257]= {1.27316, 14.0047}

In[254]:=
```
1 - CDF[ChiSquareDistribution[Length[d_small] - 2]][14.004719612621194`]
```

Out[254]= 0.232732

In[216]:=
```
Show[LogPlot[{f_gauss,small[x], f_poisson,small[x]},
  {x, 0, 16}, PlotStyle → {Automatic, {Thick, Dotted, Red}},
  ImageSize → Full, PlotLabel → "Vergleich Gauß-Fit und Poisson-Fit",
  AxesLabel → {"Zerfälle pro 100 ms", "Anzahl"}, PlotRange → {{0, 16}, Full}]
  (*,ListLogPlot[Map[(If[#[[2]]==0,{#[[1]],0.017},#]&),data_large],
  Joined→True,InterpolationOrder→0]*)]
```

Out[216]=