243 Thermisches Rauschen

```
Needs["ErrorBarPlots`"]
```

Messung des Frequenzgangs des Verstärkers und des Bandfilters

```
{Decrease, ΔDecrease} = 0.001 * {1, 0.002}
```

$\{0.001, 2. \times 10^{-6}\}$

```
Uin = Quantity[0.2, "Volts"]
```

0.2 V

```
ν = Quantity[100, "Hertz"]
```

100 Hz

Einstellungen
Vertical Scale: 5 db/div
V-Range: 0.1 V
Frequency Range: 1 MHz
Frequency Start: 100 Hz
Frequency Step Size: 20%
Automatic Voltage Scale

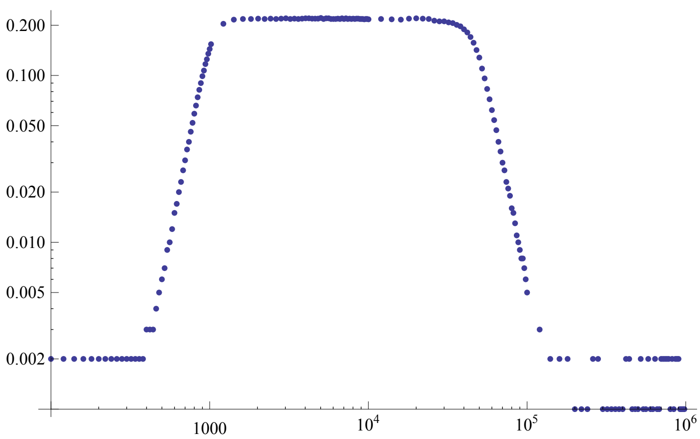
```
data2 = StringSplit@
    StringSplit[Import["/Users/jannis/Dropbox/uniself/AP2/2.2/243 Thermisches
        Rauschen/Meßwerte.txt"], "\n"][[2 ;;]] // ToExpression;
```

```
Uout = data2[[All, 2]];
```

```
Max[Uout]
```

0.221

```
ListLogLogPlot[data2[[All, {1, 2}]]]
```



```
ν = Quantity[#, "Hertz"] &@data2[[All, 1]];
```

```
{g, Δg} = Table[
  Transpose[{QuantityMagnitude[ν], 1/Decrease * Uout/QuantityMagnitude[Uin] * f}],
  {f, {1, ΔDecrease/Decrease}}];
```

```
ListLogLogPlot[{g, g[[20 ;; 137]]}, ImageSize → Full, Joined → True];
```

```
{fit, Δfit} = With[{start = 20, end = 137}, {g[[start ;; end]], Δg[[start ;; end]]}];
```

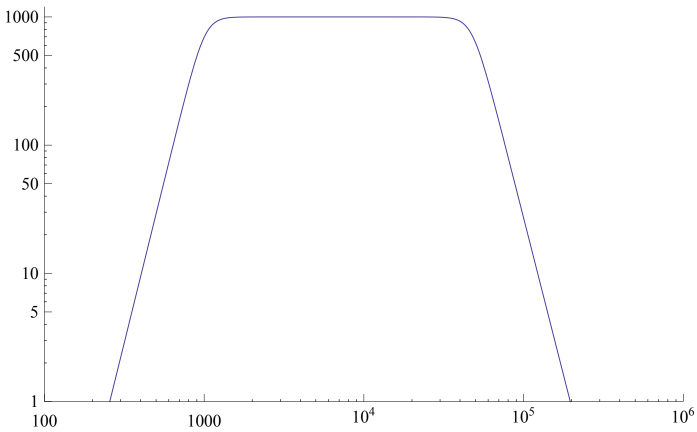
```
fml = With[{V = 1000}, NonlinearModelFit[fit,
  V/(Sqrt[1 + 1/(f/Ω1)^(2 n1)] Sqrt[1 + (f/Ω2)^(2 n2)]), {{Ω1, 1000}, {Ω2, 50 000}, {n1, 5}, {n2, 5}},
  f, VarianceEstimatorFunction → (1 &), Weights → 1/Δfit[[All, 2]]^2]]
```

FittedModel[$\frac{1000}{\sqrt{1+\frac{\ll 23 \gg}{f^{\ll 19 \gg}}}\sqrt{1+\ll 24 \gg f^{\ll 18 \gg}}}$]

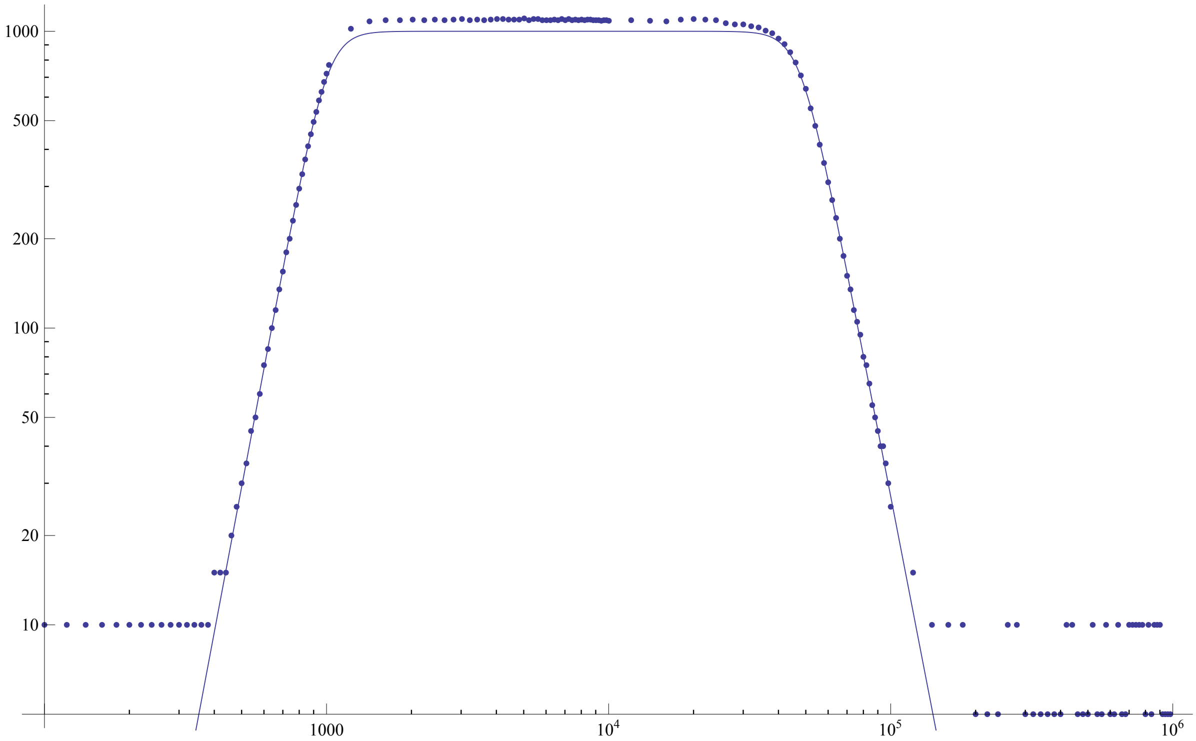
```
fml["ParameterTable"]
```

	Estimate	Standard Error	t-Statistic	P-Value
Ω1	1007.15	0.180157	5590.43	$8.077110892460950\times10^{-312}$
Ω2	47 918.2	9.70686	4936.53	1.16395×10^{-305}
n1	5.04965	0.00209607	2409.1	3.83834×10^{-270}
n2	4.90442	0.00202145	2426.19	1.71473×10^{-270}

```
LogLogPlot[fml[x], {x, 10, 1 000 000}, PlotRange → {{100, 1 000 000}, {1, 1200}}]
```



```
Show[ListLogLogPlot[g], LogLogPlot[fml[x], {x, 300, 150 000}], ImageSize → Full]
```




```
frequenzgang[f_] = Normal[fml]
```

$$\frac{1000}{\sqrt{1+\frac{2.13402\times10^{30}}{f^{10.0993}}}\ \sqrt{1+1.22938\times10^{-46}\ f^{9.80885}}}$$

```
B = Quantity[NIntegrate[frequenzgang[f]^2, {f, 0, 1 000 000}], "Hertz"]
```

4.77238×10^{10} Hz

```
ΔB = 0.02 * B
```

9.54476×10^{8} Hz

Messung der Rausch-Spannung als Funktion des ohmschen Widerstandes

```
data = {
  {Quantity[ 5 * {1, 0.005}, "Kiloohms"],
   Quantity[{2.4320, 0.00851 / Sqrt[107]}, "Millivolts"]},
  {Quantity[10 * {1, 0.005}, "Kiloohms"],
   Quantity[{3.1366, 0.0115 / Sqrt[106]}, "Millivolts"]},
  {Quantity[15 * {1, 0.005}, "Kiloohms"], Quantity[{3.7130, 0.0119 / Sqrt[105]},
    "Millivolts"]}, {Quantity[20 * {1, 0.005}, "Kiloohms"],
   Quantity[{4.2193, 0.0161 / Sqrt[104]}, "Millivolts"]},
  {Quantity[25 * {1, 0.005}, "Kiloohms"],
   Quantity[{4.6808, 0.0174 / Sqrt[103]}, "Millivolts"]},
  {Quantity[30 * {1, 0.005}, "Kiloohms"],
   Quantity[{5.0737, 0.0237 / Sqrt[186]}, "Millivolts"]}
 }
```

```
{{{5 kΩ, 0.025 kΩ}, {2.432 mV, 0.000822693 mV}},
 {{10 kΩ, 0.05 kΩ}, {3.1366 mV, 0.00111698 mV}},
 {{15 kΩ, 0.075 kΩ}, {3.713 mV, 0.00116132 mV}},
 {{20 kΩ, 0.1 kΩ}, {4.2193 mV, 0.00157873 mV}},
 {{25 kΩ, 0.125 kΩ}, {4.6808 mV, 0.00171447 mV}},
 {{30 kΩ, 0.15 kΩ}, {5.0737 mV, 0.00173777 mV}}}
```

$\{T_{Raum}, \Delta T_{Raum}\}$ =

```
 {UnitConvert[Quantity[24.7, "Celsius"], "Kelvins"], Quantity[0.1, "Kelvins"]}
```

```
{297.85 K, 0.1 K}
```

Verstärkerrauschen:

$\{U_V, \Delta U_V\}$ = `Quantity[{1.3981, 0.00467 / Sqrt[101]}, "Millivolts"]`

```
{1.3981 mV, 0.000464682 mV}
```

$\{TR, TU_{aus}\}$ = `Transpose[data]`

```
{{{5 kΩ, 0.025 kΩ}, {10 kΩ, 0.05 kΩ}, {15 kΩ, 0.075 kΩ},
  {20 kΩ, 0.1 kΩ}, {25 kΩ, 0.125 kΩ}, {30 kΩ, 0.15 kΩ}},
 {{2.432 mV, 0.000822693 mV}, {3.1366 mV, 0.00111698 mV},
  {3.713 mV, 0.00116132 mV}, {4.2193 mV, 0.00157873 mV},
  {4.6808 mV, 0.00171447 mV}, {5.0737 mV, 0.00173777 mV}}}
```

```
{R, ΔR} = Transpose[TR]
```

```
{{5 kΩ, 10 kΩ, 15 kΩ, 20 kΩ, 25 kΩ, 30 kΩ},
 {0.025 kΩ, 0.05 kΩ, 0.075 kΩ, 0.1 kΩ, 0.125 kΩ, 0.15 kΩ}}
```

```
{Uaus, ΔUaus} = Transpose[TUaus]
```

{{2.432 mV, 3.1366 mV, 3.713 mV, 4.2193 mV, 4.6808 mV, 5.0737 mV}, {0.000822693 mV, 0.00111698 mV, 0.00116132 mV, 0.00157873 mV, 0.00171447 mV, 0.00173777 mV}}

$G = U_{aus}^2 - U_V^2$

$\{3.95994\,mV^2, 7.88358\,mV^2, 11.8317\,mV^2, 15.8478\,mV^2, 19.9552\,mV^2, 23.7877\,mV^2\}$

$\Delta G = \sqrt{(2\,U_{aus}\,\Delta U_{aus})^2 + (2\,U_V\,\Delta U_V)^2}$

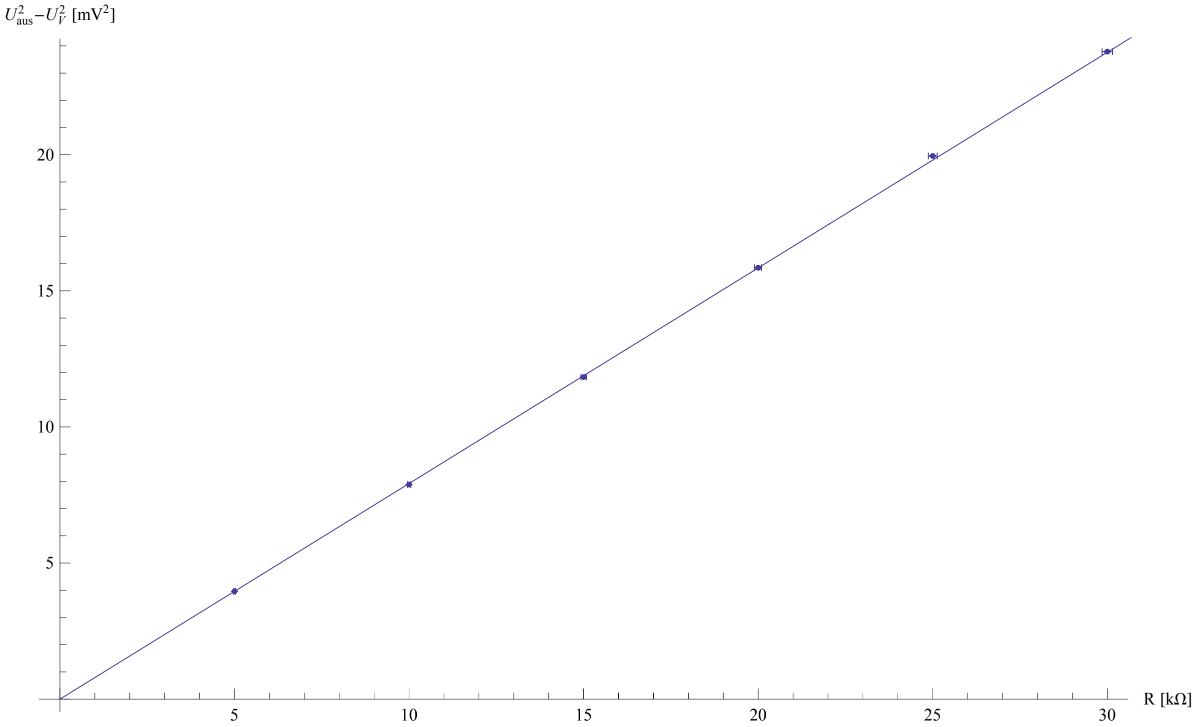
$\{0.00420725\,mV^2, 0.00712648\,mV^2, 0.00872131\,mV^2, 0.0133855\,mV^2, 0.0161027\,mV^2, 0.0176816\,mV^2\}$

```
gplot = ErrorListPlot[Table[
    With[
     {
      r = R[[i]] // QuantityMagnitude,
      Δr = ΔR[[i]] // QuantityMagnitude,
      g = G[[i]] // QuantityMagnitude,
      Δg = ΔG[[i]] // QuantityMagnitude
     },
     {{r, g}, ErrorBar[Δr, Δg]}], {i, Length[G]}],
   ImageSize → Full, AxesLabel → {"R  [kΩ]", "U_aus^2-U_V^2  [mV^2]"}];
```

```
lmf = LinearModelFit[Transpose[QuantityMagnitude /@ {R, G}],
  x, x, VarianceEstimatorFunction → (1 &),
  Weights → 1 / (QuantityMagnitude[ΔR]^2 + QuantityMagnitude[ΔG]^2),
  IncludeConstantBasis → False]
```

FittedModel[0.792109 x]

```
Show[gplot, Plot[lmf[x], {x, 0, 100}]]
```




```
lmf["RSquared"]
```

0.999983

$\{\{c, \Delta c\}\} = \text{Quantity}\left[\text{lmf["ParameterTableEntries"][[All, 1 ;; 2]]}, \frac{\text{"Millivolts"}^2}{\text{"Kiloohms"}}\right]$

$\{\{0.792109\ \text{mV}^2/\text{k}\Omega, 0.00205994\ \text{mV}^2/\text{k}\Omega\}\}$

$\{k, \Delta k_{stat}\} = \text{UnitConvert}\left[\frac{\{c, \Delta c\}}{4\, T_{Raum}\, B}, \frac{\text{"Joules"}}{\text{"Kelvins"}}\right]$

$\{1.39313 \times 10^{-23}\ \text{J/K}, 3.62295 \times 10^{-26}\ \text{J/K}\}$

$\Delta k_{syst} = \text{UnitConvert}\left[\sqrt{\left(\frac{c\, \Delta B}{4\, T_{Raum}\, B^2}\right)^2 + \left(\frac{c\, \Delta T_{Raum}}{4\, T_{Raum}{}^2\, B}\right)^2}, \frac{\text{"Joules"}}{\text{"Kelvins"}}\right]$

$2.78666 \times 10^{-25}\ \text{J/K}$