Gekoppelte Pendel

Ungekoppelte Schwingung

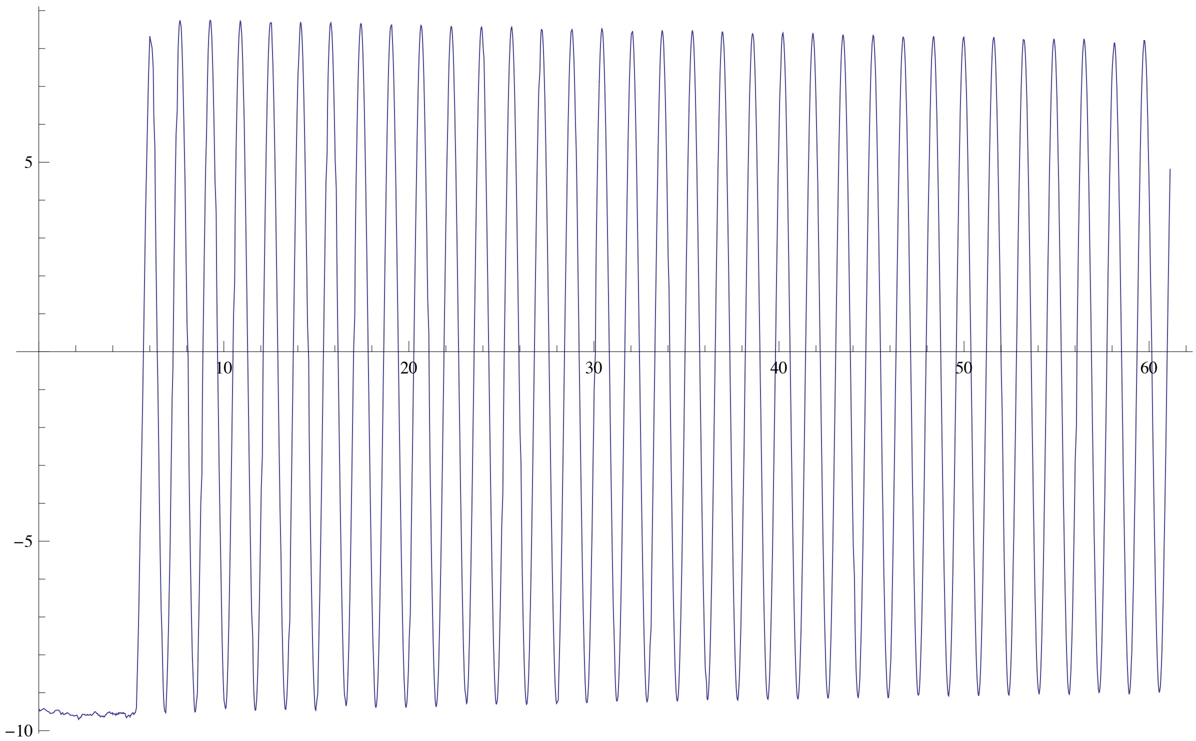
Wir importieren zunächst die Daten der ungekoppelten Pendel:

```
{time, p1, p2} = Transpose[Import[
    "/Users/jannis/Dropbox/uniself/AP2/221 gekoppelte Pendel/ungekoppelt.txt",
    "Table"]];
```

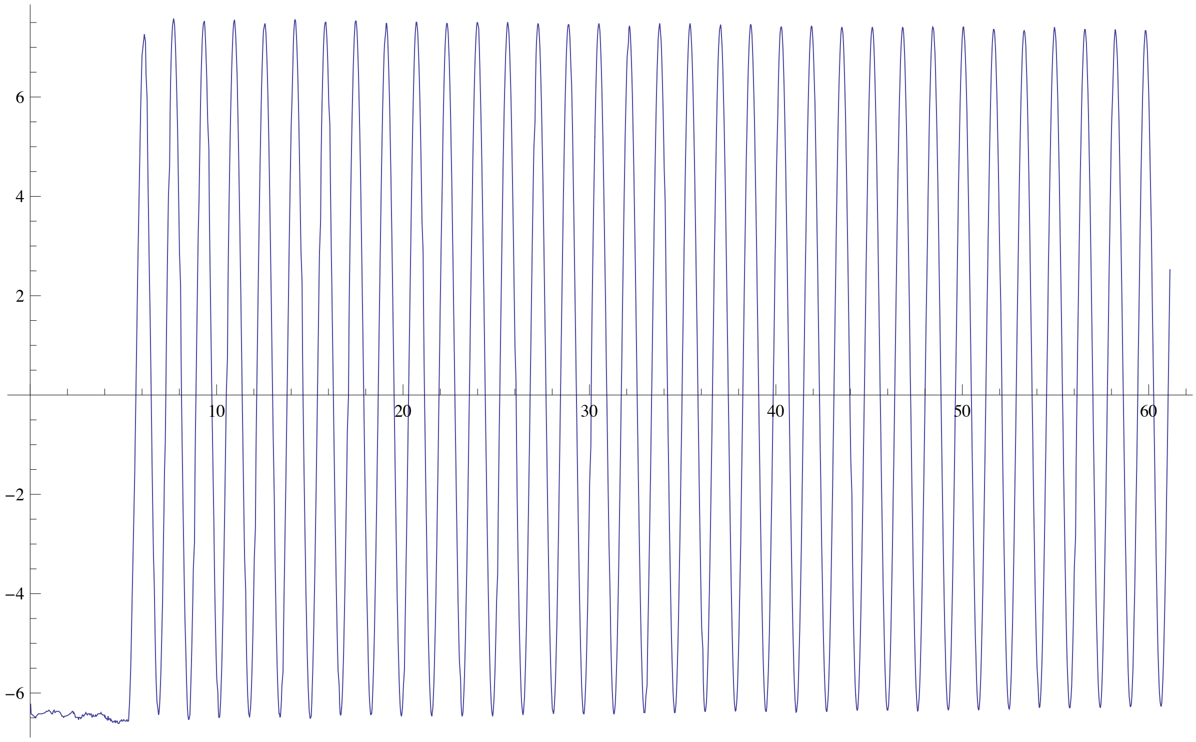
```
ungekoppelt1 = Transpose[{time, p1}];
```

```
ungekoppelt2 = Transpose[{time, p2}];
```

```
plotu1 = ListPlot[ungekoppelt1, Joined → True, ImageSize → Full]
```



```
plotu2 = ListPlot[ungekoppelt2, Joined → True, ImageSize → Full]
```




```
maxQ[{{x1_, y1_}, {x2_, y2_}, {x3_, y3_}}] := y1 < y2 && y2 ≥ y3;
```

```
minQ[{{x1_, y1_}, {x2_, y2_}, {x3_, y3_}}] := y1 ≥ y2 && y2 < y3;
```

Durch Messung der Abstände der Peaks und Mittlung finden wir die Periodendauer und daraus die Winkelfrequenz:

```
maxu1 = Select[
  Map[#[[2]] &, Select[Partition[ungekoppelt1, 3, 1], maxQ[#] &]], #[[2]] > 0 &]
```

```
{{5.998, 8.323}, {7.634, 8.744}, {9.258, 8.744}, {10.888, 8.722},
 {12.514, 8.678}, {14.143, 8.7}, {15.786, 8.678}, {17.408, 8.656},
 {19.07, 8.633}, {20.653, 8.611}, {22.29, 8.589}, {23.929, 8.567},
 {25.557, 8.567}, {27.16, 8.5}, {28.816, 8.5}, {30.424, 8.522}, {32.09, 8.456},
 {33.686, 8.478}, {35.318, 8.478}, {36.926, 8.434}, {38.577, 8.389},
 {40.216, 8.411}, {41.836, 8.411}, {43.46, 8.367}, {45.088, 8.345},
 {46.712, 8.3}, {48.36, 8.323}, {49.99, 8.3}, {51.61, 8.3}, {53.201, 8.234},
 {54.854, 8.256}, {56.479, 8.256}, {58.115, 8.145}, {59.726, 8.212}}
```

```
minu1 = Select[
  Map[#[[2]] &, Select[Partition[ungekoppelt1, 3, 1], minQ[#] &]], #[[1]] > 6 &]
```

```
{{6.861, -9.526}, {8.461, -9.504}, {10.093, -9.415},
 {11.719, -9.46}, {13.35, -9.437}, {14.969, -9.46}, {16.602, -9.349},
 {18.215, -9.393}, {19.841, -9.393}, {21.471, -9.371},
 {23.114, -9.282}, {24.727, -9.304}, {26.357, -9.304}, {27.982, -9.282},
 {29.633, -9.26}, {31.246, -9.215}, {32.866, -9.238}, {34.507, -9.215},
 {36.138, -9.193}, {37.773, -9.193}, {39.403, -9.171}, {41.025, -9.171},
 {42.656, -9.149}, {44.28, -9.127}, {45.92, -9.127}, {47.546, -9.06},
 {49.171, -9.082}, {50.788, -9.082}, {52.429, -9.06}, {54.06, -9.038},
 {55.692, -9.038}, {57.292, -8.993}, {58.922, -9.038}, {60.549, -8.993}}
```

```
periods_u1 = Quantity[
  Join[Differences[max_u1[[All, 1]]], Differences[min_u1[[All, 1]]]], "Seconds"]
```

{1.636 s, 1.624 s, 1.63 s, 1.626 s, 1.629 s, 1.643 s, 1.622 s, 1.662 s, 1.583 s, 1.637 s, 1.639 s, 1.628 s, 1.603 s, 1.656 s, 1.608 s, 1.666 s, 1.596 s, 1.632 s, 1.608 s, 1.651 s, 1.639 s, 1.62 s, 1.624 s, 1.628 s, 1.624 s, 1.648 s, 1.63 s, 1.62 s, 1.591 s, 1.653 s, 1.625 s, 1.636 s, 1.611 s, 1.6 s, 1.632 s, 1.626 s, 1.631 s, 1.619 s, 1.633 s, 1.613 s, 1.626 s, 1.63 s, 1.643 s, 1.613 s, 1.63 s, 1.625 s, 1.651 s, 1.613 s, 1.62 s, 1.641 s, 1.631 s, 1.635 s, 1.63 s, 1.622 s, 1.631 s, 1.624 s, 1.64 s, 1.626 s, 1.625 s, 1.617 s, 1.641 s, 1.631 s, 1.632 s, 1.6 s, 1.63 s, 1.627 s}

```
T_u1 = Mean[periods_u1]; ΔT_u1 = StandardDeviation[periods_u1];
```

Die Winkelfrequenz des ersten Pendels ist mit Fehler:

$\omega_{u1} = \frac{2\pi}{T_{u1}}$ // N

3.8606 per second

$\Delta\omega_{u1} = \frac{2\pi\,\Delta T_{u1}}{T_{u1}^2}$ // N

0.0371773 per second

```
max_u2 = Select[
  Map[#[[2]] &, Select[Partition[ungekoppelt_2, 3, 1], maxQ[#] &]], #[[2]] > 0 &]
```

{{6.123, 7.263}, {7.694, 7.579}, {9.346, 7.53}, {10.964, 7.554}, {12.601, 7.482}, {14.193, 7.554}, {15.838, 7.506}, {17.459, 7.53}, {19.096, 7.482}, {20.705, 7.506}, {22.344, 7.482}, {23.981, 7.506}, {25.61, 7.506}, {27.239, 7.482}, {28.87, 7.457}, {30.506, 7.482}, {32.143, 7.433}, {33.766, 7.482}, {35.399, 7.482}, {37.035, 7.457}, {38.636, 7.457}, {40.273, 7.409}, {41.918, 7.433}, {43.544, 7.409}, {45.171, 7.409}, {46.794, 7.409}, {48.413, 7.409}, {50.043, 7.409}, {51.67, 7.36}, {53.317, 7.336}, {54.94, 7.409}, {56.563, 7.36}, {58.201, 7.36}, {59.811, 7.336}}

```
min_u2 = Select[
  Map[#[[2]] &, Select[Partition[ungekoppelt_2, 3, 1], minQ[#] &]], #[[1]] > 6 &]
```

{{6.888, -6.442}, {8.512, -6.539}, {10.144, -6.491}, {11.769, -6.491}, {13.4, -6.491}, {15.019, -6.515}, {16.654, -6.442}, {18.266, -6.442}, {18.316, -6.418}, {19.917, -6.467}, {21.549, -6.467}, {23.166, -6.467}, {24.807, -6.467}, {26.436, -6.442}, {28.072, -6.418}, {29.711, -6.418}, {31.327, -6.418}, {32.973, -6.394}, {34.589, -6.394}, {36.219, -6.369}, {37.826, -6.369}, {39.486, -6.369}, {41.078, -6.394}, {42.739, -6.369}, {44.365, -6.345}, {46.003, -6.345}, {47.6, -6.369}, {49.255, -6.345}, {50.873, -6.345}, {52.517, -6.321}, {54.13, -6.296}, {55.779, -6.296}, {57.404, -6.296}, {59.032, -6.272}, {60.668, -6.272}}

```
periods_u2 = Quantity[Select[Join[Differences[max_u2[[2 ;;, 1]]],
    Differences[min_u2[[All, 1]]]], # > 1 &], "Seconds"]
```

{1.652 s, 1.618 s, 1.637 s, 1.592 s, 1.645 s, 1.621 s, 1.637 s, 1.609 s, 1.639 s, 1.637 s, 1.629 s, 1.629 s, 1.631 s, 1.636 s, 1.637 s, 1.623 s, 1.633 s, 1.636 s, 1.601 s, 1.637 s, 1.645 s, 1.626 s, 1.627 s, 1.623 s, 1.619 s, 1.63 s, 1.627 s, 1.647 s, 1.623 s, 1.623 s, 1.638 s, 1.61 s, 1.624 s, 1.632 s, 1.625 s, 1.631 s, 1.619 s, 1.635 s, 1.612 s, 1.601 s, 1.632 s, 1.617 s, 1.641 s, 1.629 s, 1.636 s, 1.639 s, 1.616 s, 1.646 s, 1.616 s, 1.63 s, 1.607 s, 1.66 s, 1.592 s, 1.661 s, 1.626 s, 1.638 s, 1.597 s, 1.655 s, 1.618 s, 1.644 s, 1.613 s, 1.649 s, 1.625 s, 1.628 s, 1.636 s}

```
T_u2 = Mean[periods_u2]; ΔT_u2 = StandardDeviation[periods_u2];
```

Die Winkelfrequenz des zweiten Pendels ist mit Fehler:

$$\omega_{u2} = \frac{2\pi}{T_{u2}} \; // \; N$$

3.85847 per second

$$\Delta\omega_{u2} = \frac{2\pi \, \Delta T_{u2}}{T_{u2}^{2}} \; // \; N$$

0.0354873 per second

Schwache Kopplung (Aufhängung oben)

Die Aufhängung der Feder befand sich hier bei

```
l_oben = Quantity[5.5, "Centimeters"]
```

5.5 cm

```
Δl_oben = Quantity[0.1, "Centimeters"]
```

0.1 cm

Wir importieren zunächst wieder die Rohdaten:

```
{time, p1, p2} =
  Transpose[Import["/Users/jannis/Dropbox/uniself/AP2/221 gekoppelte
      Pendel/gekoppelt-oben-antisymmetrisch.txt", "Table"]];
```

```
gekoppelt_oben,asym,1 = Transpose[{time, p1}];
```

```
gekoppelt_oben,asym,2 = Transpose[{time, p2}];
```

```
{time, p1, p2} =
  Transpose[Import["/Users/jannis/Dropbox/uniself/AP2/221 gekoppelte
      Pendel/gekoppelt-oben-symmetrisch.txt", "Table"]];
```

```
gekoppelt_oben,sym,1 = Transpose[{time, p1}];
```

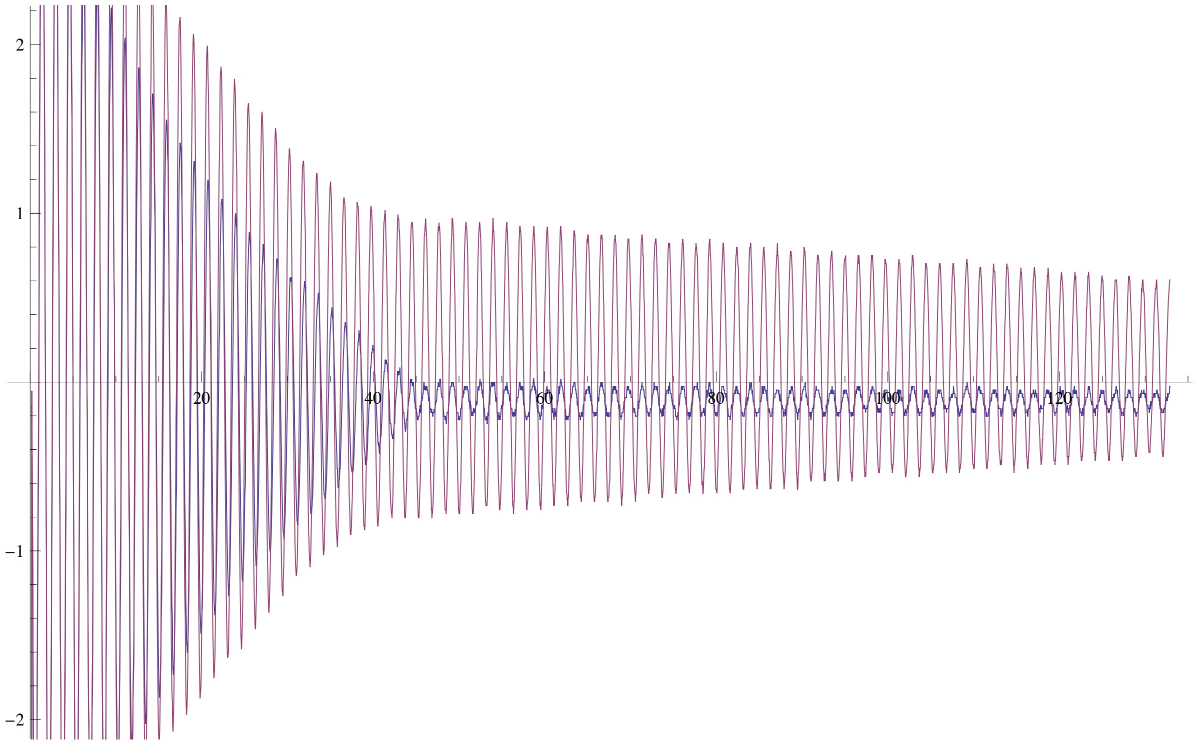
```
gekoppelt_oben,sym,2 = Transpose[{time, p2}];
```

```
{time, p1, p2} =
  Transpose[Import["/Users/jannis/Dropbox/uniself/AP2/221 gekoppelte
      Pendel/gekoppelt-oben-schwebung.txt", "Table"]];
```

```
gekoppelt_oben,schweb,1 = Transpose[{time, p1}];
```

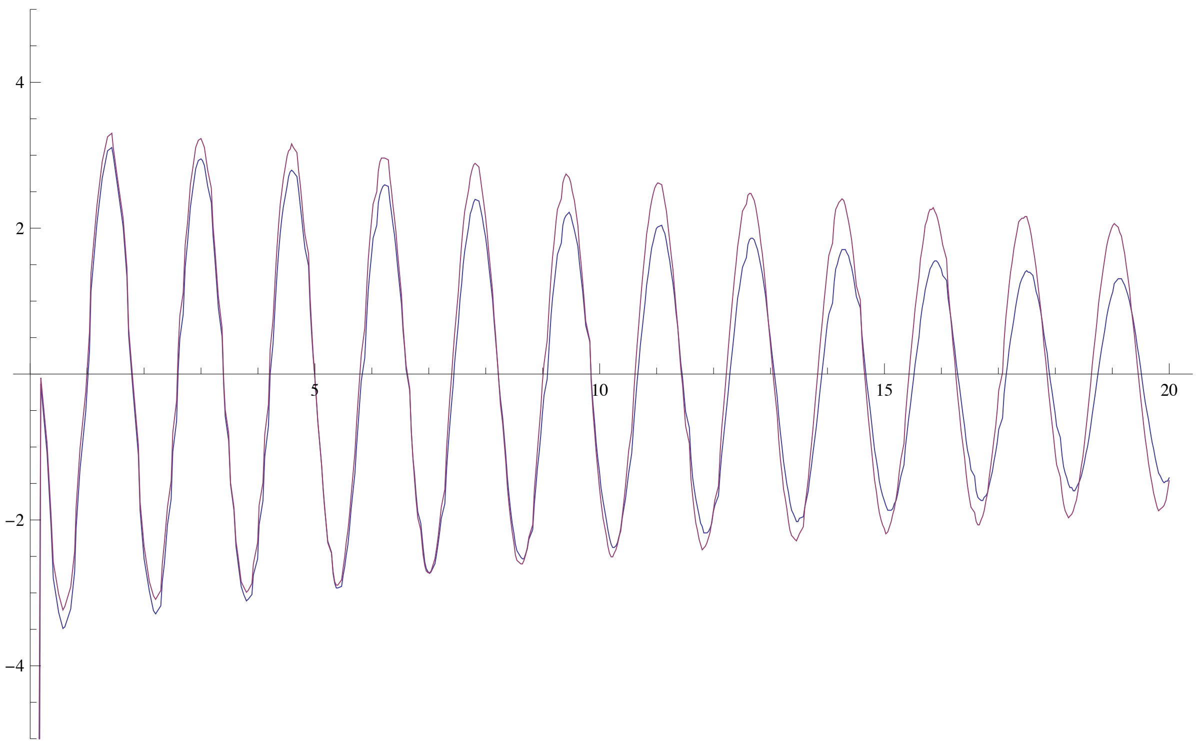
```
gekoppelt_oben,schweb,2 = Transpose[{time, p2}];
```

```
ListPlot[{ gekoppelt_{oben,sym,1}, gekoppelt_{oben,sym,2}}, Joined → True, ImageSize → Full]
```



Aus Gründen der Datenqualität (Größe der Peaks) nutzen wir die ersten 20 Sekunden:

```
gekoppelt_{oben,sym,1} = Select[gekoppelt_{oben,sym,1}, #[[1]] ≤ 20 &];

gekoppelt_{oben,sym,2} = Select[gekoppelt_{oben,sym,2}, #[[1]] ≤ 20 &];

ListPlot[{ gekoppelt_{oben,sym,1}, gekoppelt_{oben,sym,2}},
 Joined → True, ImageSize → Full, PlotRange → {Full, {-5, 5}}]
```



Nun bestimmen wir die Frequenz $\omega_{\text{oben},1}$ anhand der Daten von beiden Pendeln. Wir nutzen die Methode von oben.

```
max_{oben,sym,1} = Select[Map[#[[2]] &,
   Select[Partition[gekoppelt_{oben,sym,1}, 3, 1], maxQ[#] &]], #[[2]] > 0 &]
```

```
{{1.434, 3.106}, {2.999, 2.95}, {4.591, 2.795}, {6.224, 2.595},
 {7.812, 2.395}, {9.464, 2.218}, {11.089, 2.04}, {12.656, 1.862},
 {14.255, 1.707}, {15.883, 1.552}, {17.498, 1.418}, {19.104, 1.307}}
```

```
min_{oben,sym,1} = Select[Map[#[[2]] &,
   Select[Partition[gekoppelt_{oben,sym,1}, 3, 1], minQ[#] &]], #[[2]] < 0 &]
```

```
{{0.575, -3.488}, {2.203, -3.288}, {3.797, -3.11},
 {5.401, -2.933}, {7.014, -2.733}, {8.671, -2.533}, {10.254, -2.378},
 {11.884, -2.178}, {13.482, -2.023}, {15.119, -1.867},
 {16.726, -1.734}, {18.341, -1.601}, {19.91, -1.49}, {19.971, -1.468}}
```

```
max_{oben,sym,2} =
 Select[Map[#[[2]] &, Select[Partition[gekoppelt_{oben,sym,2}, 3, 1], maxQ[#] &]],
   #[[2]] > 0 &][[2 ;;]]
```

```
{{2.999, 3.229}, {4.591, 3.156}, {6.169, 2.962}, {7.812, 2.889}, {9.409, 2.743},
 {11.022, 2.622}, {12.63, 2.476}, {14.255, 2.403}, {15.8, 2.257},
 {15.857, 2.281}, {17.413, 2.136}, {17.468, 2.16}, {19.035, 2.063}}
```

```
min_{oben,sym,2} = Select[Map[#[[2]] &,
   Select[Partition[gekoppelt_{oben,sym,2}, 3, 1], minQ[#] &]], #[[2]] < 0 &]
```

```
{{0.575, -3.235}, {2.203, -3.089}, {3.797, -2.992},
 {5.401, -2.894}, {7.014, -2.724}, {8.643, -2.603},
 {10.228, -2.506}, {11.8, -2.409}, {13.455, -2.287}, {15.024, -2.19},
 {16.665, -2.068}, {18.232, -1.971}, {19.813, -1.874}}
```

```
periods_{oben,1} = Select[Flatten[Differences /@
     (Part[#, All, 1] & /@ {max_{oben,sym,1}, min_{oben,sym,1}, max_{oben,sym,2}, min_{oben,sym,2}})],
   # > 1 &] // Quantity[#, "Seconds"] &
```

```
{1.565 s, 1.592 s, 1.633 s, 1.588 s, 1.652 s, 1.625 s, 1.567 s, 1.599 s, 1.628 s,
 1.615 s, 1.606 s, 1.628 s, 1.594 s, 1.604 s, 1.613 s, 1.657 s, 1.583 s, 1.63 s,
 1.598 s, 1.637 s, 1.607 s, 1.615 s, 1.569 s, 1.592 s, 1.578 s, 1.643 s, 1.597 s,
 1.613 s, 1.608 s, 1.625 s, 1.545 s, 1.556 s, 1.567 s, 1.628 s, 1.594 s, 1.604 s,
 1.613 s, 1.629 s, 1.585 s, 1.572 s, 1.655 s, 1.569 s, 1.641 s, 1.567 s, 1.581 s}
```

```
T_{oben,1} = Mean[periods_{oben,1}]; ΔT_{oben,1} = StandardDeviation[periods_{oben,1}];
```

$$\omega_{\text{oben},1} = \frac{2\pi}{T_{\text{oben},1}} \text{ // N}$$

3.9179 per second

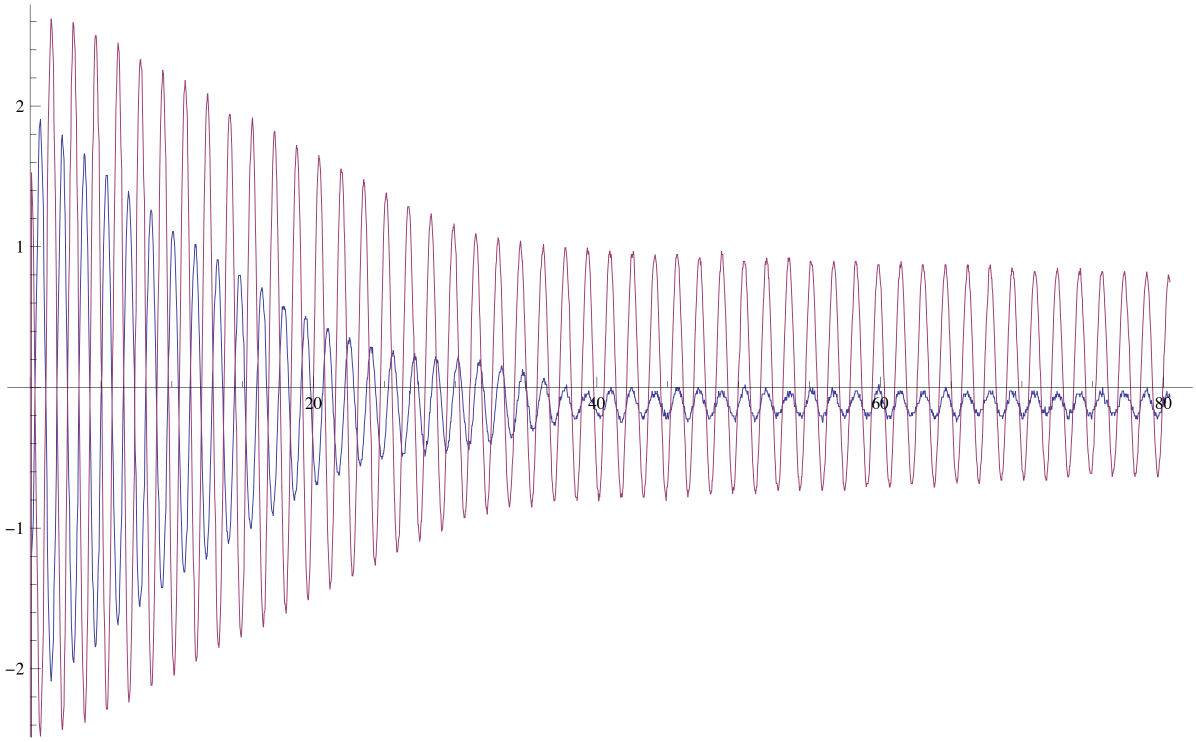
$$\Delta\omega_{\text{oben},1} = \frac{2\pi\,\Delta T_{\text{oben},1}}{T_{\text{oben},1}^2} \text{ // N}$$

0.0686098 per second

Das Ergebnis stimmt innerhalb der Fehlergrenzen mit den Frequenzen ohne Kopplung überein.

Antisymmetrische Schwingung

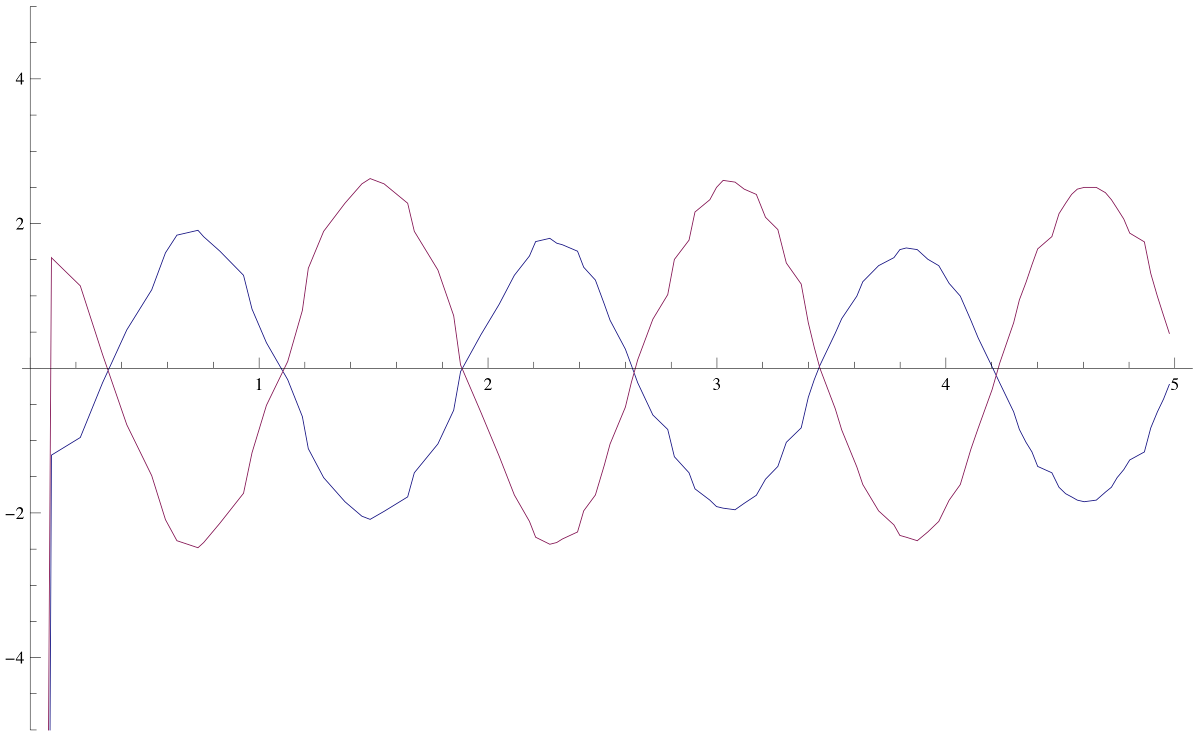
```
ListPlot[{ gekoppelt_{oben,asym,1}, gekoppelt_{oben,asym,2}}, Joined → True, ImageSize → Full]
```



Es muß eine Störung dazu geführt haben, daß nach den ersten Perioden die Schwingungen sich gegeneinander verschieben. Daher betrachten wir nur die ersten 5 Sekunden:

```
gekoppelt_{oben,asym,1} = Select[gekoppelt_{oben,asym,1}, #[[1]] ≤ 5 &];

gekoppelt_{oben,asym,2} = Select[gekoppelt_{oben,asym,2}, #[[1]] ≤ 5 &];
```

```
ListPlot[{ gekoppelt_{oben,asym,1}, gekoppelt_{oben,asym,2}},
 Joined → True, ImageSize → Full, PlotRange → {Full, {-5, 5}}]
```



Nun bestimmen wir die Frequenz $\omega_{oben,2}$ anhand der Daten von beiden Pendeln.

```
max_{oben,asym,1} = Select[Map[#[[2]] &,
   Select[Partition[gekoppelt_{oben,asym,1}, 3, 1], maxQ[#] &]], #[[2]] > 0 &]
{{0.733, 1.907}, {2.27, 1.796}, {3.827, 1.663}}
```

```
min_{oben,asym,1} = Select[Map[#[[2]] &,
   Select[Partition[gekoppelt_{oben,asym,1}, 3, 1], minQ[#] &]], #[[2]] < 0 &]
{{1.485, -2.089}, {3.079, -1.956}, {4.604, -1.845}}
```

```
max_{oben,asym,2} =
 Select[Map[#[[2]] &, Select[Partition[gekoppelt_{oben,asym,2}, 3, 1], maxQ[#] &]],
   #[[2]] > 0 &][[2 ;;]]
{{1.485, 2.622}, {3.027, 2.597}, {4.604, 2.5}}
```

```
min_{oben,asym,2} = Select[Map[#[[2]] &,
   Select[Partition[gekoppelt_{oben,asym,2}, 3, 1], minQ[#] &]], #[[2]] < 0 &]
{{0.733, -2.481}, {2.27, -2.433}, {3.875, -2.384}}
```

```
periods_{oben,2} = Select[Flatten[Differences /@
      (Part[#, All, 1] & /@ {max_{oben,asym,1}, min_{oben,asym,1}, max_{oben,asym,2}, min_{oben,asym,2}})],
   # > 1 &] // Quantity[#, "Seconds"] &
{1.537 s, 1.557 s, 1.594 s, 1.525 s, 1.542 s, 1.577 s, 1.537 s, 1.605 s}
```

```
T_{oben,2} = Mean[periods_{oben,2}]; ΔT_{oben,2} = StandardDeviation[periods_{oben,2}];
```

$$\omega_{oben,2} = \frac{2\,\pi}{T_{oben,2}} \;//\; N$$

4.02962 per second

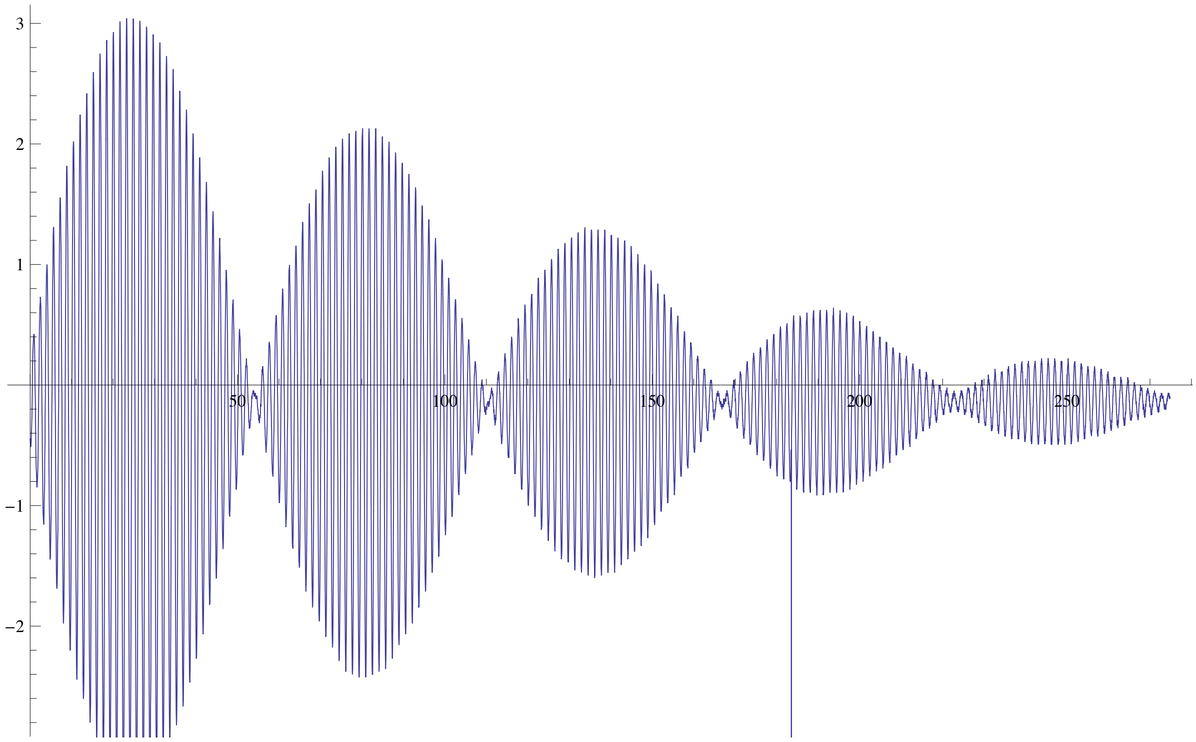
$$\Delta\omega_{oben,2} = \frac{2\,\pi\,\Delta T_{oben,2}}{T_{oben,2}{}^{2}} \;//\; N$$

0.0761736 per second

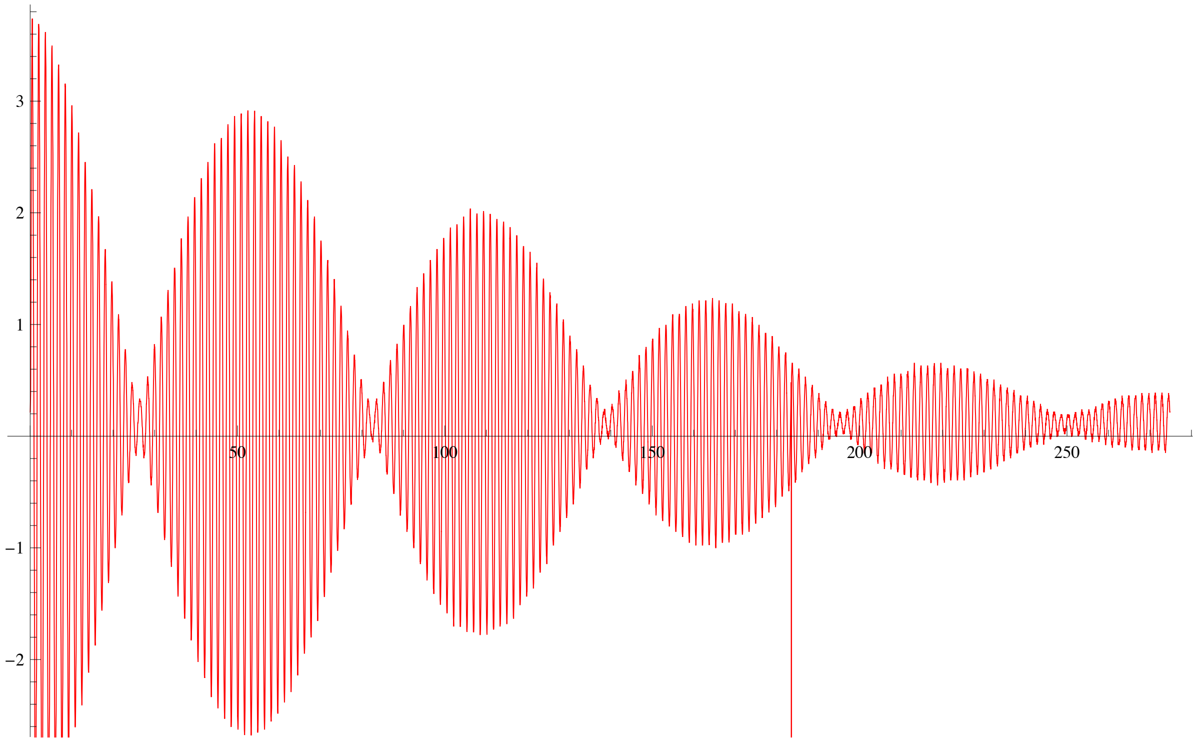
Wie erwartet, ist diese Frequenz etwas größer.

Schwebung

```
ListPlot[gekoppelt_{oben,schweb,1}, Joined → True, ImageSize → Full]
```



```
ListPlot[gekoppelt_{oben,schweb,2}, Joined → True, ImageSize → Full, PlotStyle → Red]
```



Da die Schwebung das Vorzeichen der Schwingung umkehrt, ist es am besten, zur Bestimmung von $\omega_{\text{oben,I}}$ nur einen Schwebungsblock zu nutzen. Wir wählen von beiden Pendeln die ersten 20 Sekunden. Zur Bestimmung der Schwebungsfrequenz $\omega_{\text{oben,II}}$ wählen wir manuell (mithilfe der graphischen Funktion »Get Coordinates«) Schwebungspeaks aus und ermitteln deren Differenzen. (Wir erhalten dann die doppelte Frequenz, da die Einhüllende ja auch Ausschläge nach Oben und Unten hat.

Zunächst ermitteln wir $\omega_{\text{oben,II}}$:

```
peaks_{oben,schweb,1} =
  {{24.56, 3.047}, {81.97, 2.133}, {137.3, 1.293}, {192.8, 0.6317}, {247.8, 0.2222}}
{{24.56, 3.047}, {81.97, 2.133}, {137.3, 1.293}, {192.8, 0.6317}, {247.8, 0.2222}}

peaks_{oben,schweb,2} = {{52.96, 2.931}, {108.5, 2.023}, {165.4, 1.241}, {220.3, 0.6513}}
{{52.96, 2.931}, {108.5, 2.023}, {165.4, 1.241}, {220.3, 0.6513}}

periods_{oben,II} = Select[Flatten[
     Differences /@ (Part[#, All, 1] & /@ {peaks_{oben,schweb,1}, peaks_{oben,schweb,2}})],
   # > 1 &] // Quantity[#, "Seconds"] &
{57.41 s, 55.33 s, 55.5 s, 55. s, 55.54 s, 56.9 s, 54.9 s}

T_{oben,II} = Mean[periods_{oben,II}]; ΔT_{oben,II} = StandardDeviation[periods_{oben,II}];
```

In diesem Schritt schreiben wir in den Zähler nur π, um die Dopplung (s. o.) aus der Rechnung zu entfernen.

$$\omega_{\text{oben,II}} = \frac{\pi}{T_{\text{oben,II}}} \; // \; N$$

```
0.0563038 per second
```

$\Delta\omega_{oben,II} = \frac{\pi\, \Delta T_{oben,II}}{T_{oben,II}^2}$ // N

0.00097752 per second

Nun bestimmen wir $\omega_{oben,I}$:

```
max_{oben,schweb,1} = Select[
  Map[#[[2]] &, Select[Partition[Select[gekoppelt_{oben,schweb,1}, #[[1]] ≤ 20 &], 3, 1],
    maxQ[#] &]], #[[2]] > 0 &]
```

{{0.868, 0.419}, {2.45, 0.73}, {4.051, 0.997}, {5.605, 1.307}, {7.192, 1.552}, {8.843, 1.818}, {10.435, 2.018}, {12.024, 2.24}, {13.633, 2.417}, {15.239, 2.595}, {16.848, 2.75}, {18.469, 2.861}}

```
min_{oben,schweb,1} = Select[
  Map[#[[2]] &, Select[Partition[Select[gekoppelt_{oben,schweb,1}, #[[1]] ≤ 20 &], 3, 1],
    minQ[#] &]], #[[2]] < 0 &]
```

{{0.118, -0.513}, {1.672, -0.846}, {3.299, -1.157}, {4.829, -1.445}, {6.442, -1.69}, {8.031, -1.978}, {9.672, -2.2}, {11.264, -2.444}, {12.859, -2.6}, {14.456, -2.8}, {16.039, -2.933}, {17.629, -3.11}, {19.251, -3.199}}

```
max_{oben,schweb,2} = Select[
  Map[#[[2]] &, Select[Partition[Select[gekoppelt_{oben,schweb,2}, #[[1]] ≤ 20 &], 3, 1],
    maxQ[#] &]], #[[2]] > 0 &]
```

{{0.514, 3.739}, {2.039, 3.691}, {3.674, 3.618}, {5.283, 3.496}, {6.871, 3.326}, {8.449, 3.156}, {10.055, 2.962}, {11.677, 2.719}, {13.246, 2.451}, {14.849, 2.208}, {16.49, 1.965}, {18.112, 1.674}, {19.673, 1.382}}

```
min_{oben,schweb,2} = Select[
  Map[#[[2]] &, Select[Partition[Select[gekoppelt_{oben,schweb,2}, #[[1]] ≤ 20 &], 3, 1],
    minQ[#] &]], #[[2]] < 0 &]
```

{{1.27, -3.478}, {2.836, -3.429}, {4.433, -3.308}, {6.089, -3.21}, {7.675, -3.016}, {9.25, -2.822}, {10.874, -2.603}, {12.471, -2.409}, {14.109, -2.117}, {15.693, -1.874}, {17.325, -1.558}, {18.885, -1.315}, {18.946, -1.291}}

```
periods_{oben,I} =
 Select[Flatten[Differences /@ (Part[#, All, 1] & /@ {max_{oben,schweb,1}, min_{oben,schweb,1},
       max_{oben,schweb,2}, min_{oben,schweb,2}})], # > 1 &] // Quantity[#, "Seconds"] &
```

{1.582 s, 1.601 s, 1.554 s, 1.587 s, 1.651 s, 1.592 s, 1.589 s, 1.609 s, 1.606 s, 1.609 s, 1.621 s, 1.554 s, 1.627 s, 1.53 s, 1.613 s, 1.589 s, 1.641 s, 1.592 s, 1.595 s, 1.597 s, 1.583 s, 1.59 s, 1.622 s, 1.525 s, 1.635 s, 1.609 s, 1.588 s, 1.578 s, 1.606 s, 1.622 s, 1.569 s, 1.603 s, 1.641 s, 1.622 s, 1.561 s, 1.566 s, 1.597 s, 1.656 s, 1.586 s, 1.575 s, 1.624 s, 1.597 s, 1.638 s, 1.584 s, 1.632 s, 1.56 s}

```
T_{oben,I} = Mean[periods_{oben,I}]; ΔT_{oben,I} = StandardDeviation[periods_{oben,I}];
```

$\omega_{oben,I} = \frac{2\pi}{T_{oben,I}}$ // N

3.93191 per second

$$\Delta\omega_{oben,I} = \frac{2\,\pi\,\Delta T_{oben,I}}{T_{oben,I}^2} \;//\; N$$

0.0727174 per second

Vergleich der Frequenzen

Aus den nun ermittelten Frequenzen für symmetrische und antisymmetrische Schwingung lassen sich die theoretischen Werte für Schwingungs- und Schwebungsfrequenzen ermitteln und so $\omega_{\text{oben,I}}$ und $\omega_{\text{oben,II}}$ – theoretisch und gemessen – vergleichen.

($\omega_{\text{oben,I}} = \frac{1}{2}(\omega_{\text{oben,1}} + \omega_{\text{oben,2}})$ und $\Delta\omega_{\text{oben,I}} = \frac{1}{2}(\Delta\omega_{\text{oben,1}} + \Delta\omega_{\text{oben,2}})$, $\omega_{\text{oben,II}} = \frac{1}{2}(\omega_{\text{oben,2}} - \omega_{\text{oben,1}})$ und $\Delta\omega_{\text{oben,II}} = \frac{1}{2}(\Delta\omega_{\text{oben,1}} + \Delta\omega_{\text{oben,2}})$)

$$\omega_{oben,I,theor} = \frac{1}{2}(\omega_{oben,1} + \omega_{oben,2})$$

3.97376 per second

$$\Delta\omega_{oben,I,theor} = \frac{1}{2}(\Delta\omega_{oben,1} + \Delta\omega_{oben,2})$$

0.0723917 per second

$$\omega_{oben,II,theor} = \frac{1}{2}(\omega_{oben,2} - \omega_{oben,1})$$

0.0558584 per second

$$\Delta\omega_{oben,II,theor} = \frac{1}{2}(\Delta\omega_{oben,2} + \Delta\omega_{oben,1})$$

0.0723917 per second

```
Grid[{{Null, Text["Theoretischer Wert"], Text["Gemessener Wert"]},
  {Text["ωoben,I"], NumberForm[ωoben,I,theor, 3] ± NumberForm[Δωoben,I,theor, 1],
   NumberForm[ωoben,I, 3] ± NumberForm[Δωoben,I, 1]},
  {Text["ωoben,II"], NumberForm[ωoben,II,theor, 1] ± NumberForm[Δωoben,II,theor, 1],
   NumberForm[ωoben,II, 3] ± NumberForm[Δωoben,II, 1]}}, Frame → All]
```

	Theoretischer Wert	Gemessener Wert
$\omega_{\text{oben,I}}$	3.97 per second ± 0.07 per second	3.93 per second ± 0.07 per second
$\omega_{\text{oben,II}}$	0.06 per second ± 0.07 per second	0.0563 per second ± 0.001 per second

Wir stellen fest, daß insbesondere der relative Fehler auf $\omega_{\text{oben,II}}$ sehr groß ist, nichtsdestoweniger alle Werte innerhalb ihrer Fehlergrenzen übereinstimmen.

Kopplungsgrad

Nun berechnen wir den Kopplungsgrad aus den gemessenen Frequenzen:

$$\kappa_{oben} = \frac{\omega_{oben,2}^2 - \omega_{oben,1}^2}{\omega_{oben,2}^2 + \omega_{oben,1}^2}$$

0.028108

Mit Gauß’scher Fehlerfortpflanzung ergibt sich

$$\Delta\kappa_{oben} = \frac{4\,\omega_{oben,1}\,\omega_{oben,2}}{\left(\omega_{oben,2}^2 + \omega_{oben,1}^2\right)^2} \sqrt{(\omega_{oben,1}\,\Delta\omega_{oben,2})^2 + (\omega_{oben,2}\,\Delta\omega_{oben,1})^2}$$

0.0257479

Auch hier ist der relative Fehler sehr groß. Wir können mit diesem Ergebnis lediglich die Größenordnung von κ einschätzen.

Mittelstarke Kopplung (Aufhängung in der Mitte)

Hinweis: Alle Rechnungen gleichen denen unter »Schwache Kopplung«, daher wurde auf redundante Kommentierung verzichtet.

Die Aufhängung der Feder befand sich hier bei

```
lmitte = Quantity[15.5, "Centimeters"]
```

```
15.5 cm
```

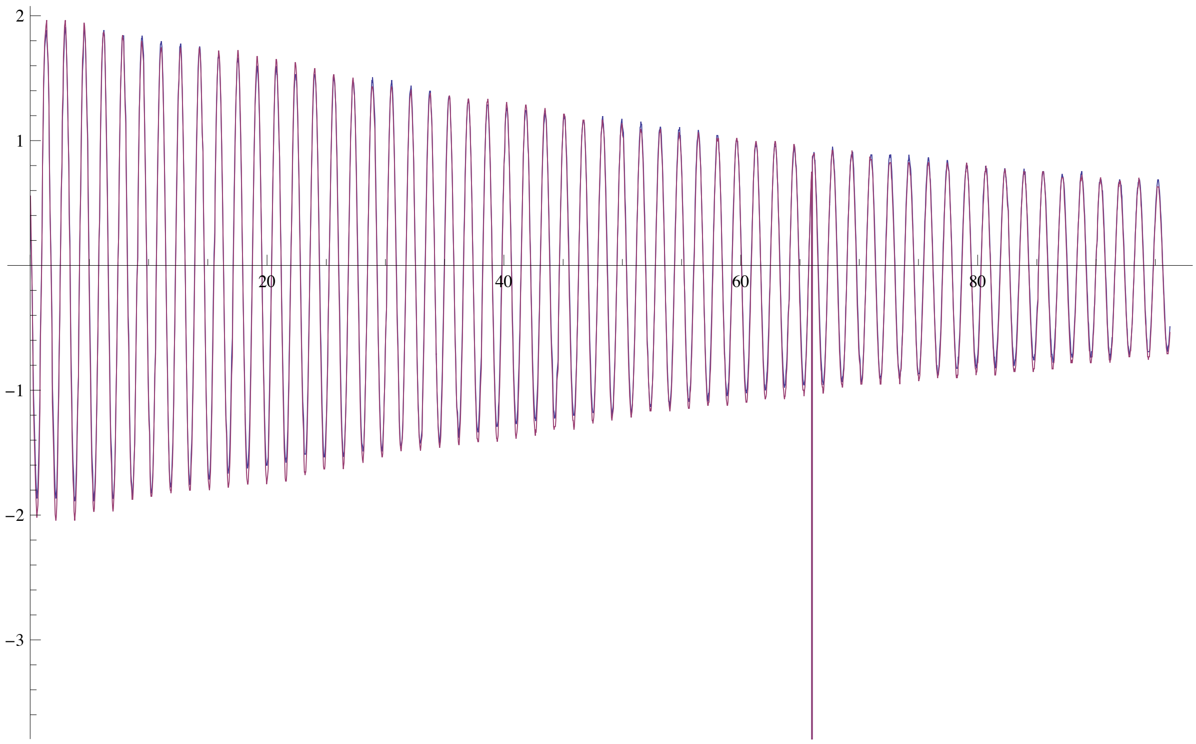
```
Δlmitte = Quantity[0.1, "Centimeters"]
```

```
0.1 cm
```

Wir importieren zunächst wieder die Rohdaten:

```
{time, p1, p2} =
  Transpose[Import["/Users/jannis/Dropbox/uniself/AP2/221 gekoppelte
      Pendel/gekoppelt-mitte-antisymmetrisch.txt", "Table"]];

gekoppeltmitte,asym,1 = Transpose[{time, p1}];

gekoppeltmitte,asym,2 = Transpose[{time, p2}];

{time, p1, p2} =
  Transpose[Import["/Users/jannis/Dropbox/uniself/AP2/221 gekoppelte
      Pendel/gekoppelt-mitte-symmetrisch.txt", "Table"]];

gekoppeltmitte,sym,1 = Transpose[{time, p1}];

gekoppeltmitte,sym,2 = Transpose[{time, p2}];

{time, p1, p2} =
  Transpose[Import["/Users/jannis/Dropbox/uniself/AP2/221 gekoppelte
      Pendel/gekoppelt-mitte-schwebung.txt", "Table"]];

gekoppeltmitte,schweb,1 = Transpose[{time, p1}];

gekoppeltmitte,schweb,2 = Transpose[{time, p2}];
```

```
ListPlot[{ gekoppelt_{mitte,sym,1}, gekoppelt_{mitte,sym,2}}, Joined → True, ImageSize → Full]
```



Wir nutzen die ersten 60 Sekunden, um den störenden Ausreißer (ein Fehler des Meßgeräts) auf einfache Weise loszuwerden:

```
gekoppelt_{mitte,sym,1} = Select[gekoppelt_{mitte,sym,1}, #[[1]] ≤ 60 &];

gekoppelt_{mitte,sym,2} = Select[gekoppelt_{mitte,sym,2}, #[[1]] ≤ 60 &];
```

Wir bestimmen $\omega_{\text{mitte},1}$.

```
max_{mitte,sym,1} = Select[Map[#[[2]] &,
   Select[Partition[gekoppelt_{mitte,sym,1}, 3, 1], maxQ[#] &]], #[[2]] > 0 &]
```

```
{{1.398, 1.885}, {2.968, 1.907}, {4.542, 1.885}, {6.165, 1.862},
 {6.219, 1.885}, {7.797, 1.84}, {9.399, 1.818}, {9.453, 1.84},
 {10.999, 1.774}, {11.075, 1.796}, {12.631, 1.751}, {12.694, 1.774},
 {14.296, 1.751}, {15.858, 1.663}, {15.914, 1.685}, {17.532, 1.663},
 {19.148, 1.596}, {20.798, 1.596}, {22.371, 1.529}, {23.986, 1.529},
 {25.647, 1.529}, {27.222, 1.463}, {28.85, 1.485}, {28.908, 1.507},
 {30.528, 1.485}, {32.164, 1.441}, {33.729, 1.396}, {35.35, 1.352},
 {36.99, 1.33}, {38.573, 1.285}, {40.2, 1.241}, {40.256, 1.263},
 {41.805, 1.241}, {43.473, 1.219}, {45.039, 1.174}, {45.102, 1.196},
 {46.675, 1.152}, {48.343, 1.196}, {49.961, 1.174}, {51.502, 1.108},
 {51.56, 1.152}, {53.133, 1.085}, {53.197, 1.108}, {54.741, 1.085},
 {54.839, 1.108}, {56.415, 1.085}, {57.973, 1.041}, {59.606, 0.997}}
```

```
min_mitte,sym,1 = Select[Map[#[[2]] &,
    Select[Partition[gekoppelt_mitte,sym,1, 3, 1], minQ[#] &]], #[[2]] < 0 &]
```

```
{{0.562, -1.867}, {2.169, -1.867}, {3.763, -1.889},
 {5.364, -1.889}, {7.023, -1.867}, {8.603, -1.845}, {8.661, -1.823},
 {10.263, -1.823}, {11.864, -1.778}, {13.494, -1.756},
 {15.136, -1.712}, {16.715, -1.667}, {18.357, -1.623},
 {20., -1.601}, {21.599, -1.579}, {23.272, -1.512}, {24.862, -1.534},
 {26.466, -1.534}, {28.107, -1.49}, {29.703, -1.49}, {31.331, -1.445},
 {32.956, -1.423}, {34.576, -1.423}, {36.182, -1.379}, {37.825, -1.334},
 {39.438, -1.29}, {41.054, -1.268}, {42.684, -1.246}, {44.298, -1.223},
 {45.951, -1.201}, {47.585, -1.179}, {49.149, -1.201}, {50.778, -1.179},
 {52.348, -1.135}, {52.416, -1.135}, {54.015, -1.135}, {55.653, -1.09},
 {57.208, -1.09}, {57.313, -1.024}, {58.848, -1.046}, {58.917, -1.024}}
```

```
max_mitte,sym,2 = Select[Map[#[[2]] &,
    Select[Partition[gekoppelt_mitte,sym,2, 3, 1], maxQ[#] &]], #[[2]] > 0 &]
```

```
{{1.398, 1.965}, {2.968, 1.965}, {4.542, 1.941}, {6.192, 1.868}, {7.797, 1.844},
 {9.399, 1.795}, {11.075, 1.747}, {12.694, 1.747}, {14.296, 1.747},
 {15.914, 1.722}, {17.532, 1.722}, {19.148, 1.674}, {20.762, 1.65},
 {22.371, 1.625}, {24.013, 1.577}, {25.616, 1.528}, {27.26, 1.504}, {28.85, 1.431},
 {30.501, 1.431}, {32.164, 1.407}, {33.729, 1.358}, {33.796, 1.382},
 {35.383, 1.358}, {36.99, 1.334}, {38.573, 1.309}, {38.641, 1.334},
 {40.231, 1.309}, {41.831, 1.285}, {43.473, 1.261}, {45.039, 1.212},
 {46.675, 1.164}, {48.303, 1.164}, {49.961, 1.139}, {51.534, 1.091},
 {53.133, 1.066}, {53.197, 1.091}, {54.741, 1.018}, {54.806, 1.066},
 {56.415, 1.066}, {57.973, 1.018}, {58.123, 1.018}, {59.644, 1.018}}
```

```
min_mitte,sym,2 = Select[Map[#[[2]] &,
    Select[Partition[gekoppelt_mitte,sym,2, 3, 1], minQ[#] &]], #[[2]] < 0 &]
```

```
{{0.562, -2.02}, {2.169, -2.044}, {3.763, -2.044},
 {5.392, -1.971}, {6.996, -1.971}, {8.661, -1.874}, {10.263, -1.85},
 {11.889, -1.825}, {13.523, -1.801}, {15.136, -1.801},
 {16.741, -1.777}, {18.387, -1.752}, {19.966, -1.752}, {21.624, -1.728},
 {23.215, -1.679}, {23.272, -1.655}, {24.862, -1.631}, {26.438, -1.631},
 {28.107, -1.582}, {29.738, -1.534}, {31.331, -1.485}, {32.956, -1.485},
 {34.576, -1.461}, {36.221, -1.436}, {37.859, -1.412}, {39.438, -1.412},
 {41.029, -1.388}, {42.652, -1.364}, {42.717, -1.339}, {44.24, -1.315},
 {44.347, -1.291}, {45.917, -1.315}, {47.518, -1.266}, {49.117, -1.242},
 {50.749, -1.218}, {52.348, -1.169}, {52.416, -1.169}, {54.015, -1.169},
 {55.653, -1.145}, {57.248, -1.121}, {57.313, -1.096}, {58.884, -1.121}}
```

```
periods_{mitte,1} = Select[Flatten[Differences /@
    (Part[#, All, 1] & /@ {max_{mitte,sym,1}, min_{mitte,sym,1}, max_{mitte,sym,2}, min_{mitte,sym,2}})],
  # > 1 &] // Quantity[#, "Seconds"] &
```

{1.57 s, 1.574 s, 1.623 s, 1.578 s, 1.602 s, 1.546 s, 1.556 s, 1.602 s, 1.562 s, 1.618 s, 1.616 s, 1.65 s, 1.573 s, 1.615 s, 1.661 s, 1.575 s, 1.628 s, 1.62 s, 1.636 s, 1.565 s, 1.621 s, 1.64 s, 1.583 s, 1.627 s, 1.549 s, 1.668 s, 1.566 s, 1.573 s, 1.668 s, 1.618 s, 1.541 s, 1.573 s, 1.544 s, 1.576 s, 1.558 s, 1.633 s, 1.607 s, 1.594 s, 1.601 s, 1.659 s, 1.58 s, 1.602 s, 1.601 s, 1.63 s, 1.642 s, 1.579 s, 1.642 s, 1.643 s, 1.599 s, 1.673 s, 1.59 s, 1.604 s, 1.641 s, 1.596 s, 1.628 s, 1.625 s, 1.62 s, 1.606 s, 1.643 s, 1.613 s, 1.616 s, 1.63 s, 1.614 s, 1.653 s, 1.634 s, 1.564 s, 1.629 s, 1.57 s, 1.599 s, 1.638 s, 1.555 s, 1.535 s, 1.57 s, 1.574 s, 1.65 s, 1.605 s, 1.602 s, 1.676 s, 1.619 s, 1.602 s, 1.618 s, 1.618 s, 1.616 s, 1.614 s, 1.609 s, 1.642 s, 1.603 s, 1.644 s, 1.59 s, 1.651 s, 1.663 s, 1.565 s, 1.587 s, 1.607 s, 1.583 s, 1.59 s, 1.6 s, 1.642 s, 1.566 s, 1.636 s, 1.628 s, 1.658 s, 1.573 s, 1.599 s, 1.544 s, 1.609 s, 1.558 s, 1.521 s, 1.607 s, 1.594 s, 1.629 s, 1.604 s, 1.665 s, 1.602 s, 1.626 s, 1.634 s, 1.613 s, 1.605 s, 1.646 s, 1.579 s, 1.658 s, 1.591 s, 1.59 s, 1.576 s, 1.669 s, 1.631 s, 1.593 s, 1.625 s, 1.62 s, 1.645 s, 1.638 s, 1.579 s, 1.591 s, 1.623 s, 1.523 s, 1.57 s, 1.601 s, 1.599 s, 1.632 s, 1.599 s, 1.599 s, 1.638 s, 1.595 s, 1.571 s}

```
T_{mitte,1} = Mean[periods_{mitte,1}]; ΔT_{mitte,1} = StandardDeviation[periods_{mitte,1}];
```

$$\omega_{mitte,1} = \frac{2\pi}{T_{mitte,1}} \;//\; N$$

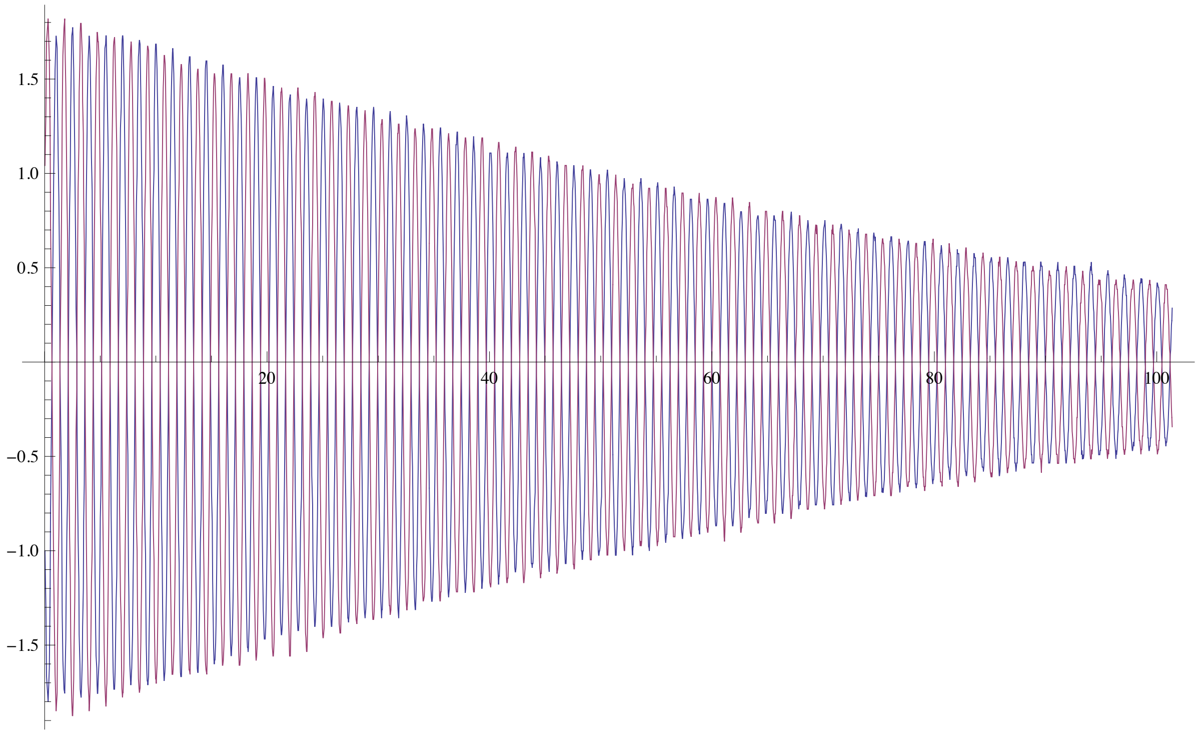
3.91078 per second

$$\Delta\omega_{mitte,1} = \frac{2\pi\,\Delta T_{mitte,1}}{T_{mitte,1}^2} \;//\; N$$

0.081318 per second

Antisymmetrische Schwingung

```
ListPlot[{ gekoppelt_{mitte,asym,1}, gekoppelt_{mitte,asym,2}},
 Joined → True, ImageSize → Full]
```



Wir nutzen wieder die ersten 60 Sekunden, um nicht durch Dämpfungseffekte ein zur symmetrischen Messung widersprüchliches Ergebnis zu erhalten.

```
gekoppelt_{mitte,asym,1} = Select[gekoppelt_{mitte,asym,1}, #[[1]] ≤ 60 &];
```

```
gekoppelt_{mitte,asym,2} = Select[gekoppelt_{mitte,asym,2}, #[[1]] ≤ 60 &];
```

Nun bestimmen wir die Frequenz $\omega_{\text{mitte},2}$ anhand der Daten von beiden Pendeln.

```
max_{mitte,asym,1} = Select[Map[#[[2]] &,
   Select[Partition[gekoppelt_{mitte,asym,1}, 3, 1], maxQ[#] &]], #[[2]] > 0 &]
```

```
{{1.032, 1.729}, {2.523, 1.774}, {4.005, 1.729}, {5.51, 1.729}, {6.999, 1.729},
 {8.522, 1.707}, {9.977, 1.685}, {11.521, 1.663}, {13.015, 1.618}, {14.51, 1.596},
 {16.017, 1.574}, {17.525, 1.507}, {19.016, 1.507}, {20.564, 1.463},
 {22.094, 1.418}, {23.552, 1.396}, {25.055, 1.396}, {26.524, 1.374},
 {28.076, 1.352}, {29.581, 1.352}, {31.072, 1.33}, {32.536, 1.307},
 {34.058, 1.263}, {35.578, 1.241}, {36.986, 1.152}, {37.084, 1.219},
 {38.572, 1.196}, {40.025, 1.108}, {41.533, 1.085}, {41.598, 1.108},
 {43.024, 1.085}, {43.094, 1.108}, {44.591, 1.085}, {46.063, 1.063},
 {47.569, 1.041}, {49.057, 1.019}, {50.605, 1.019}, {52.11, 0.974},
 {53.548, 0.952}, {53.618, 0.974}, {55.059, 0.93}, {55.124, 0.952},
 {56.541, 0.886}, {56.601, 0.93}, {58.052, 0.863}, {59.568, 0.863}}
```

```
minmitte,asym,1 = Select[Map[#[[2]] &,
   Select[Partition[gekoppeltmitte,asym,1, 3, 1], minQ[#] &]], #[[2]] < 0 &]
```

{{0.313, -1.801}, {1.808, -1.756}, {3.282, -1.778}, {4.774, -1.756},
{6.287, -1.734}, {7.748, -1.712}, {9.277, -1.712}, {10.747, -1.69},
{12.299, -1.667}, {13.795, -1.645}, {15.255, -1.601}, {15.315, -1.579},
{16.79, -1.556}, {18.297, -1.534}, {19.823, -1.468}, {21.309, -1.445},
{22.829, -1.423}, {24.31, -1.401}, {25.819, -1.401}, {27.323, -1.379},
{28.812, -1.357}, {30.296, -1.357}, {30.359, -1.312}, {31.837, -1.357},
{33.35, -1.312}, {34.838, -1.268}, {36.324, -1.246}, {37.819, -1.223},
{39.316, -1.201}, {40.813, -1.179}, {40.881, -1.135}, {42.345, -1.112},
{43.838, -1.09}, {43.904, -1.068}, {45.367, -1.112}, {46.884, -1.068},
{48.36, -1.046}, {48.433, -1.001}, {49.84, -1.024}, {49.903, -1.001},
{51.386, -1.024}, {52.865, -1.024}, {52.93, -0.979}, {54.373, -1.001},
{55.844, -0.957}, {55.942, -0.913}, {57.364, -0.935}, {58.89, -0.913}}

```
maxmitte,asym,2 = Select[Map[#[[2]] &,
   Select[Partition[gekoppeltmitte,asym,2, 3, 1], maxQ[#] &]], #[[2]] > 0 &]
```

{{0.313, 1.82}, {1.808, 1.82}, {3.229, 1.795}, {4.716, 1.747},
{6.287, 1.722}, {7.779, 1.698}, {9.25, 1.674}, {10.747, 1.625},
{12.272, 1.577}, {13.764, 1.552}, {15.255, 1.528}, {16.76, 1.528},
{18.268, 1.528}, {19.757, 1.504}, {21.242, 1.431}, {21.309, 1.455},
{22.773, 1.455}, {24.245, 1.407}, {24.31, 1.431}, {25.782, 1.382},
{27.291, 1.358}, {28.812, 1.334}, {30.265, 1.261}, {30.327, 1.285},
{31.742, 1.212}, {31.806, 1.261}, {33.292, 1.236}, {34.807, 1.236},
{36.324, 1.212}, {37.819, 1.188}, {39.316, 1.188}, {40.813, 1.164},
{42.275, 1.115}, {42.345, 1.139}, {43.807, 1.115}, {43.863, 1.115},
{45.299, 1.091}, {46.789, 1.042}, {48.299, 1.018}, {48.399, 1.042},
{49.84, 0.993}, {51.358, 0.993}, {52.797, 0.921}, {52.899, 0.945},
{54.308, 0.921}, {55.808, 0.921}, {57.306, 0.896}, {58.858, 0.896}}

```
minmitte,asym,2 = Select[Map[#[[2]] &,
   Select[Partition[gekoppeltmitte,asym,2, 3, 1], minQ[#] &]], #[[2]] < 0 &]
```

{{1.032, -1.85}, {2.523, -1.874}, {4.005, -1.85},
{5.51, -1.825}, {6.999, -1.777}, {7.055, -1.752}, {8.522, -1.752},
{10.035, -1.704}, {11.521, -1.655}, {13.075, -1.655},
{14.583, -1.655}, {16.017, -1.607}, {17.551, -1.607}, {19.016, -1.582},
{20.564, -1.558}, {22.094, -1.558}, {23.586, -1.534}, {25.055, -1.461},
{26.617, -1.436}, {28.076, -1.388}, {29.581, -1.364}, {31.072, -1.339},
{32.574, -1.315}, {34.058, -1.266}, {35.578, -1.266}, {35.641, -1.218},
{37.122, -1.218}, {38.629, -1.218}, {40.107, -1.194}, {41.598, -1.169},
{41.661, -1.145}, {43.094, -1.169}, {44.591, -1.145}, {44.656, -1.121},
{46.063, -1.121}, {47.633, -1.096}, {49.155, -1.048}, {50.637, -1.023},
{52.151, -0.999}, {53.548, -0.975}, {53.643, -0.975}, {55.086, -0.975},
{55.157, -0.951}, {56.672, -0.926}, {58.129, -0.926}, {59.64, -0.902}}

```
periods_{mitte,2} =
 Select[Flatten[Differences /@ (Part[#, All, 1] & /@ {max_{mitte,asym,1}, min_{mitte,asym,1},
       max_{mitte,asym,2}, min_{mitte,asym,2}})], # > 1 &] // Quantity[#, "Seconds"] &
```

{1.491 s, 1.482 s, 1.505 s, 1.489 s, 1.523 s, 1.455 s, 1.544 s, 1.494 s, 1.495 s,
1.507 s, 1.508 s, 1.491 s, 1.548 s, 1.53 s, 1.458 s, 1.503 s, 1.469 s, 1.552 s,
1.505 s, 1.491 s, 1.464 s, 1.522 s, 1.52 s, 1.408 s, 1.488 s, 1.453 s, 1.508 s,
1.426 s, 1.497 s, 1.472 s, 1.506 s, 1.488 s, 1.548 s, 1.505 s, 1.438 s, 1.441 s,
1.417 s, 1.451 s, 1.516 s, 1.495 s, 1.474 s, 1.492 s, 1.513 s, 1.461 s, 1.529 s,
1.47 s, 1.552 s, 1.496 s, 1.46 s, 1.475 s, 1.507 s, 1.526 s, 1.486 s, 1.52 s, 1.481 s,
1.509 s, 1.504 s, 1.489 s, 1.484 s, 1.478 s, 1.513 s, 1.488 s, 1.486 s, 1.495 s,
1.497 s, 1.497 s, 1.464 s, 1.493 s, 1.463 s, 1.517 s, 1.476 s, 1.407 s, 1.483 s,
1.479 s, 1.443 s, 1.471 s, 1.422 s, 1.526 s, 1.495 s, 1.421 s, 1.487 s, 1.571 s,
1.492 s, 1.471 s, 1.497 s, 1.525 s, 1.492 s, 1.491 s, 1.505 s, 1.508 s, 1.489 s,
1.485 s, 1.464 s, 1.472 s, 1.472 s, 1.509 s, 1.521 s, 1.453 s, 1.415 s, 1.486 s,
1.515 s, 1.517 s, 1.495 s, 1.497 s, 1.497 s, 1.462 s, 1.462 s, 1.436 s, 1.49 s,
1.51 s, 1.441 s, 1.518 s, 1.439 s, 1.409 s, 1.5 s, 1.498 s, 1.552 s, 1.491 s, 1.482 s,
1.505 s, 1.489 s, 1.467 s, 1.513 s, 1.486 s, 1.554 s, 1.508 s, 1.434 s, 1.534 s,
1.465 s, 1.548 s, 1.53 s, 1.492 s, 1.469 s, 1.562 s, 1.459 s, 1.505 s, 1.491 s,
1.502 s, 1.484 s, 1.52 s, 1.481 s, 1.507 s, 1.478 s, 1.491 s, 1.433 s, 1.497 s, 1.407 s,
1.57 s, 1.522 s, 1.482 s, 1.514 s, 1.397 s, 1.443 s, 1.515 s, 1.457 s, 1.511 s}

$T_{mitte,2}$ = Mean[$periods_{mitte,2}$]; $\Delta T_{mitte,2}$ = StandardDeviation[$periods_{mitte,2}$];

$$\omega_{mitte,2} = \frac{2\pi}{T_{mitte,2}} \text{ // N}$$

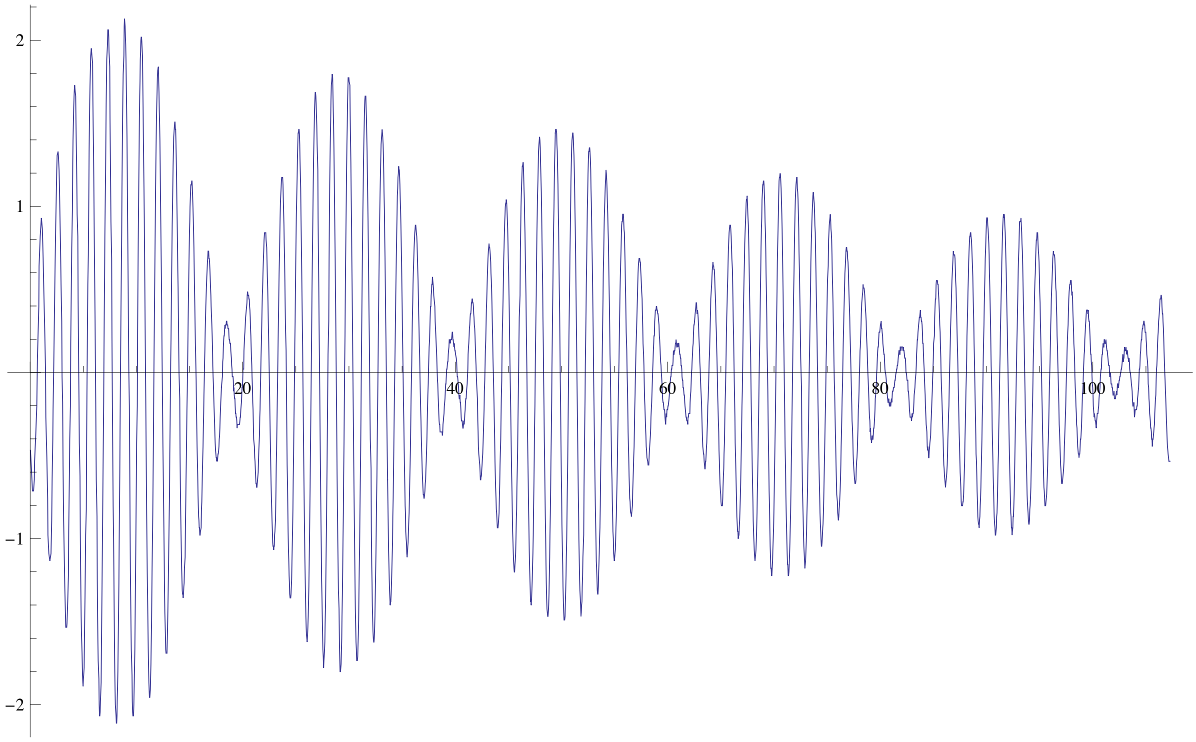
4.21975 per second

$$\Delta\omega_{mitte,2} = \frac{2\pi\,\Delta T_{mitte,2}}{T_{mitte,2}^2} \text{ // N}$$

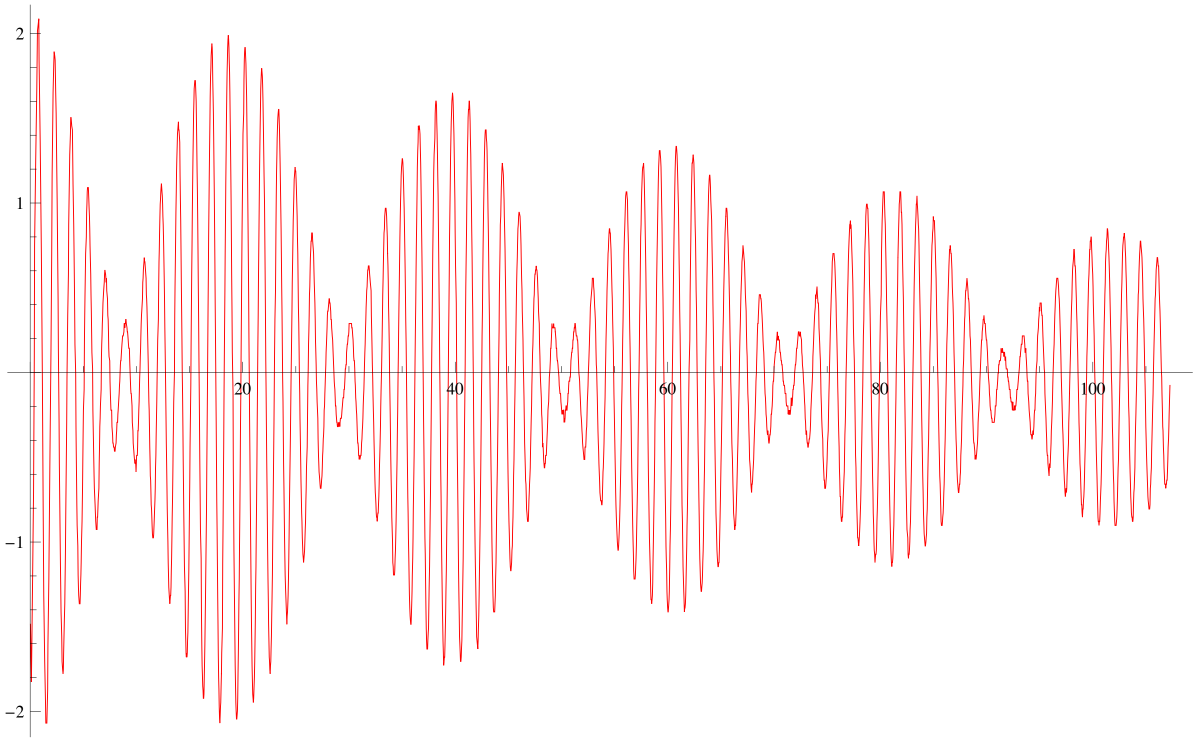
0.0968923 per second

Schwebung

```
ListPlot[gekoppelt_{mitte,schweb,1}, Joined → True, ImageSize → Full]
```




```
ListPlot[gekoppelt_{mitte,schweb,2}, Joined → True, ImageSize → Full, PlotStyle → Red]
```



Da die Schwebung das Vorzeichen der Schwingung umkehrt, ist es am besten, zur Bestimmung von $\omega_{\text{mitte},I}$ nur einen Schwebungsblock zu nutzen. Wir wählen vom ersten Pendeln die ersten 15 Sekunden und vom zweiten Pendel die Sekunden 10 bis 25. Zur Bestimmung der Schwebungsfrequenz $\omega_{\text{mitte,II}}$ wählen wir manuell (mithilfe der graphischen Funktion »Get Coordinates«) Schwebungspeaks aus und ermitteln deren Differenzen, und erhalten wieder das Doppelte der Frequenz wegen der Orientierung der Einhüllenden.

Zunächst ermitteln wir $\omega_{\text{mitte,II}}$:

```
peaks_{mitte,schweb,1} =
  {{8.869, 2.119}, {29.25, 1.799}, {50.35, 1.448}, {70.97, 1.189}, {91.71, 0.9527}}
{{8.869, 2.119}, {29.25, 1.799}, {50.35, 1.448}, {70.97, 1.189}, {91.71, 0.9527}}
```

```
peaks_{mitte,schweb,2} =
  {{18.76, 1.988}, {39.98, 1.644}, {60.84, 1.323}, {81.58, 1.076}, {101.8, 0.8447}}
{{18.76, 1.988}, {39.98, 1.644}, {60.84, 1.323}, {81.58, 1.076}, {101.8, 0.8447}}
```

```
periods_{mitte,II} = Select[Flatten[
    Differences /@ (Part[#, All, 1] & /@ {peaks_{mitte,schweb,1}, peaks_{mitte,schweb,2}})],
   # > 1 &] // Quantity[#, "Seconds"] &
{20.381 s, 21.1 s, 20.62 s, 20.74 s, 21.22 s, 20.86 s, 20.74 s, 20.22 s}
```

```
T_{mitte,II} = Mean[periods_{mitte,II}]; ΔT_{mitte,II} = StandardDeviation[periods_{mitte,II}];
```

$\omega_{\text{mitte,II}} = \frac{\pi}{T_{\text{mitte,II}}}$ // N

0.151511 per second

$\Delta\omega_{\text{mitte,II}} = \frac{\pi\,\Delta T_{\text{mitte,II}}}{T_{\text{mitte,II}}^2}$ // N

0.00245144 per second

Nun bestimmen wir $\omega_{\text{mitte,I}}$:

```
max_{mitte,schweb,1} = Select[Map[#[[2]] &, Select[Partition[
     Select[gekoppelt_{mitte,schweb,1}, #[[1]] ≤ 15 &], 3, 1], maxQ[#] &]], #[[2]] > 0 &]
{{1.055, 0.93}, {2.628, 1.33}, {4.188, 1.729}, {5.756, 1.951}, {7.306, 2.062},
 {8.879, 2.129}, {10.448, 2.018}, {12.066, 1.84}, {13.605, 1.507}}
```

```
min_{mitte,schweb,1} = Select[Map[#[[2]] &, Select[Partition[
     Select[gekoppelt_{mitte,schweb,1}, #[[1]] ≤ 15 &], 3, 1], minQ[#] &]], #[[2]] < 0 &]
{{0.318, -0.713}, {1.851, -1.135}, {3.438, -1.534},
 {4.983, -1.889}, {6.554, -2.067}, {8.141, -2.111}, {9.724, -2.067},
 {11.255, -1.956}, {12.874, -1.69}, {14.401, -1.357}}
```

```
max_{mitte,schweb,2} = Select[Map[#[[2]] &,
   Select[Partition[Select[gekoppelt_{mitte,schweb,2}, 10 ≤ #[[1]] && #[[1]] ≤ 25 &],
     3, 1], maxQ[#] &]], #[[2]] > 0 &]
{{10.74, 0.678}, {12.351, 1.115}, {13.952, 1.479},
 {15.495, 1.722}, {17.096, 1.941}, {18.647, 1.99}, {20.207, 1.917},
 {21.785, 1.795}, {23.372, 1.552}, {24.935, 1.212}}
```

```
min_{mitte,schweb,2} = Select[Map[#[[2]] &,
    Select[Partition[Select[gekoppelt_{mitte,schweb,2}, 10 ≤ #[[1]] && #[[1]] ≤ 25 &],
      3, 1], minQ[#] &]], #[[2]] < 0 &]
```

```
{{10.083, -0.489}, {10.167, -0.343}, {11.6, -0.975},
 {13.139, -1.364}, {13.224, -1.315}, {14.699, -1.679},
 {14.753, -1.679}, {16.327, -1.922}, {17.852, -2.068},
 {19.444, -2.044}, {21.005, -1.947}, {22.587, -1.777}, {24.156, -1.485}}
```

```
periods_{mitte,I} =
 Select[Flatten[Differences /@ (Part[#, All, 1] & /@ {max_{mitte,schweb,1}, min_{mitte,schweb,1},
        max_{mitte,schweb,2}, min_{mitte,schweb,2}})], # > 1 &] // Quantity[#, "Seconds"] &
```

```
{1.573 s, 1.56 s, 1.568 s, 1.55 s, 1.573 s, 1.569 s, 1.618 s, 1.539 s, 1.533 s,
 1.587 s, 1.545 s, 1.571 s, 1.587 s, 1.583 s, 1.531 s, 1.619 s, 1.527 s, 1.611 s,
 1.601 s, 1.543 s, 1.601 s, 1.551 s, 1.56 s, 1.578 s, 1.587 s, 1.563 s, 1.433 s,
 1.539 s, 1.475 s, 1.574 s, 1.525 s, 1.592 s, 1.561 s, 1.582 s, 1.569 s}
```

```
T_{mitte,I} = Mean[periods_{mitte,I}]; ΔT_{mitte,I} = StandardDeviation[periods_{mitte,I}];
```

$$\omega_{mitte,I} = \frac{2\pi}{T_{mitte,I}} \text{ // N}$$

4.02194 per second

$$\Delta\omega_{mitte,I} = \frac{2\pi\,\Delta T_{mitte,I}}{T_{mitte,I}^2} \text{ // N}$$

0.0956055 per second

Vergleich der Frequenzen

$$\omega_{mitte,I,theor} = \frac{1}{2}\left(\omega_{mitte,1} + \omega_{mitte,2}\right)$$

4.06527 per second

$$\Delta\omega_{mitte,I,theor} = \frac{1}{2}\left(\Delta\omega_{mitte,1} + \Delta\omega_{mitte,2}\right)$$

0.0891052 per second

$$\omega_{mitte,II,theor} = \frac{1}{2}\left(\omega_{mitte,2} - \omega_{mitte,1}\right)$$

0.154486 per second

$$\Delta\omega_{mitte,II,theor} = \frac{1}{2}\left(\Delta\omega_{mitte,2} + \Delta\omega_{mitte,1}\right)$$

0.0891052 per second

```
Grid[{{Null, Text["Theoretischer Wert"], Text["Gemessener Wert"]},
  {Text["ω_mitte,I"], NumberForm[ω_mitte,I,theor, 3] ± NumberForm[Δω_mitte,I,theor, 1],
   NumberForm[ω_mitte,I, 3] ± NumberForm[Δω_mitte,I, 1]},
  {Text["ω_mitte,II"], NumberForm[ω_mitte,II,theor, 2] ± NumberForm[Δω_mitte,II,theor, 1],
   NumberForm[ω_mitte,II, 3] ± NumberForm[Δω_mitte,II, 1]}}, Frame → All]
```

	Theoretischer Wert	Gemessener Wert
$\omega_{\text{mitte,I}}$	4.07 per second ± 0.09 per second	4.02 per second ± 0.1 per second
$\omega_{\text{mitte,II}}$	0.15 per second ± 0.09 per second	0.152 per second ± 0.002 per second

Kopplungsgrad

$$\varkappa_{\text{mitte}} = \frac{\omega_{\text{mitte},2}^2 - \omega_{\text{mitte},1}^2}{\omega_{\text{mitte},2}^2 + \omega_{\text{mitte},1}^2}$$

0.0758934

$$\Delta\varkappa_{\text{mitte}} = \frac{4\,\omega_{\text{mitte},1}\,\omega_{\text{mitte},2}}{\left(\omega_{\text{mitte},2}^2 + \omega_{\text{mitte},1}^2\right)^2} \sqrt{(\omega_{\text{mitte},1}\,\Delta\omega_{\text{mitte},2})^2 + (\omega_{\text{mitte},2}\,\Delta\omega_{\text{mitte},1})^2}$$

0.0307989

Starke Kopplung (Aufhängung unten)

Die Aufhängung der Feder befand sich hier bei

```
l_unten = Quantity[30.5, "Centimeters"]
```

30.5 cm

```
Δl_unten = Quantity[0.1, "Centimeters"]
```

0.1 cm

Wir importieren zunächst wieder die Rohdaten:

```
{time, p1, p2} =
  Transpose[Import["/Users/jannis/Dropbox/uniself/AP2/221 gekoppelte
      Pendel/gekoppelt-unten-antisymmetrisch.txt", "Table"]];
```

```
gekoppelt_unten,asym,1 = Transpose[{time, p1}];
```

```
gekoppelt_unten,asym,2 = Transpose[{time, p2}];
```

```
{time, p1, p2} =
  Transpose[Import["/Users/jannis/Dropbox/uniself/AP2/221 gekoppelte
      Pendel/gekoppelt-unten-symmetrisch.txt", "Table"]];
```

```
gekoppelt_unten,sym,1 = Transpose[{time, p1}];
```

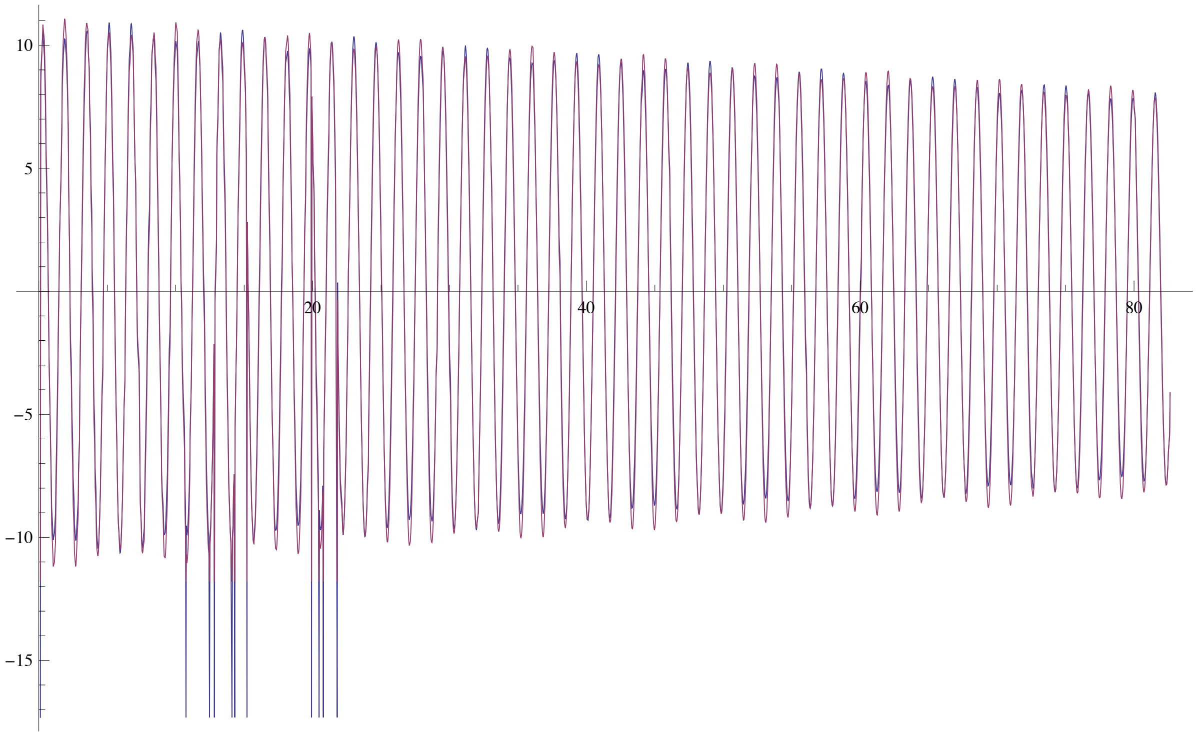
```
gekoppelt_unten,sym,2 = Transpose[{time, p2}];
```

```
{time, p1, p2} =
  Transpose[Import["/Users/jannis/Dropbox/uniself/AP2/221 gekoppelte
      Pendel/gekoppelt-unten-schwebung.txt", "Table"]];
```

```
gekoppelt_unten,schweb,1 = Transpose[{time, p1}];
```

```
gekoppelt_unten,schweb,2 = Transpose[{time, p2}];
```

```
ListPlot[{ gekoppelt_{unten,sym,1}, gekoppelt_{unten,sym,2}}, Joined → True, ImageSize → Full]
```



Da hier gerade am Anfang einige störende Ausreißer aufgetreten sind, nutzen wir die Sekunden 25 bis 80.

```
gekoppelt_{unten,sym,1} = Select[gekoppelt_{unten,sym,1}, 25 ≤ #[[1]] && #[[1]] ≤ 80 &];

gekoppelt_{unten,sym,2} = Select[gekoppelt_{unten,sym,2}, 25 ≤ #[[1]] && #[[1]] ≤ 80 &];
```

Wir bestimmen $\omega_{\text{unten},1}$.

```
max_{unten,sym,1} = Select[Map[#[[2]] &,
   Select[Partition[gekoppelt_{unten,sym,1}, 3, 1], maxQ[#] &]], #[[2]] > 0 &]
```

```
{{26.247, 9.699}, {27.881, 9.566}, {29.536, 9.766}, {31.163, 9.988},
 {32.774, 9.877}, {34.386, 9.499}, {36.029, 9.277}, {37.649, 9.388},
 {39.3, 9.677}, {40.894, 9.632}, {42.514, 9.299}, {44.18, 8.966}, {45.786, 9.033},
 {47.397, 9.277}, {49.015, 9.344}, {50.646, 9.1}, {52.277, 8.744}, {53.927, 8.7},
 {55.537, 8.922}, {57.14, 9.033}, {58.779, 8.878}, {60.416, 8.522},
 {62.063, 8.389}, {63.686, 8.567}, {65.288, 8.722}, {66.899, 8.611},
 {68.539, 8.3}, {70.176, 8.056}, {71.808, 8.189}, {73.449, 8.389},
 {75.024, 8.345}, {76.647, 8.056}, {78.292, 7.834}, {79.912, 7.834}}
```

```
min_{unten,sym,1} = Select[Map[#[[2]] &,
   Select[Partition[gekoppelt_{unten,sym,1}, 3, 1], minQ[#] &]], #[[2]] < 0 &]
```

```
{{25.45, -9.615}, {27.063, -9.282}, {28.715, -9.349},
 {30.359, -9.637}, {31.967, -9.704}, {33.578, -9.437},
 {35.219, -9.038}, {36.847, -9.038}, {38.466, -9.26}, {40.119, -9.304},
 {41.69, -9.215}, {43.321, -8.816}, {44.962, -8.705}, {46.608, -8.86},
 {48.23, -9.082}, {49.848, -8.993}, {51.471, -8.638}, {53.105, -8.416},
 {54.73, -8.527}, {56.349, -8.749}, {57.978, -8.727}, {59.607, -8.416},
 {61.248, -8.128}, {62.884, -8.172}, {64.488, -8.416}, {66.14, -8.372},
 {67.707, -8.216}, {69.334, -7.906}, {71.002, -7.861}, {72.649, -7.994},
 {74.246, -8.15}, {75.829, -7.994}, {77.489, -7.661}, {79.104, -7.528}}
```

```
max_unten,sym,2 = Select[Map[#[[2]] &,
   Select[Partition[gekoppelt_unten,sym,2, 3, 1], maxQ[#] &]], #[[2]] > 0 &]
```

```
{{26.278, 10.227}, {27.881, 10.252}, {29.505, 9.936}, {31.163, 9.523},
 {32.774, 9.547}, {34.413, 9.839}, {36.029, 9.96}, {37.649, 9.717},
 {39.269, 9.328}, {40.894, 9.207}, {42.547, 9.45}, {44.18, 9.62}, {45.753, 9.45},
 {47.397, 9.085}, {49.015, 8.867}, {50.646, 9.037}, {52.277, 9.255},
 {53.887, 9.231}, {55.51, 8.842}, {57.14, 8.599}, {58.779, 8.648}, {60.416, 8.891},
 {62.063, 8.964}, {63.64, 8.648}, {65.288, 8.332}, {66.924, 8.332},
 {68.539, 8.575}, {70.176, 8.624}, {71.774, 8.405}, {73.411, 8.089},
 {75.024, 7.968}, {76.673, 8.211}, {78.292, 8.356}, {79.912, 8.186}}
```

```
min_unten,sym,2 = Select[Map[#[[2]] &,
   Select[Partition[gekoppelt_unten,sym,2, 3, 1], minQ[#] &]], #[[2]] < 0 &]
```

```
{{25.484, -10.184}, {27.063, -10.33}, {28.715, -10.209},
 {30.327, -9.844}, {31.967, -9.626}, {33.605, -9.747},
 {35.219, -10.014}, {36.847, -9.99}, {38.466, -9.626}, {40.119, -9.213},
 {41.723, -9.383}, {43.359, -9.65}, {44.962, -9.698}, {46.575, -9.358},
 {48.23, -9.067}, {49.848, -9.042}, {51.471, -9.31}, {53.105, -9.383},
 {54.73, -9.188}, {56.349, -8.824}, {57.978, -8.702}, {59.607, -8.945},
 {61.248, -9.091}, {62.849, -8.945}, {64.457, -8.581}, {66.14, -8.338},
 {67.735, -8.556}, {69.366, -8.799}, {70.976, -8.702}, {72.615, -8.338},
 {74.246, -8.119}, {75.869, -8.192}, {77.489, -8.386}, {79.104, -8.435}}
```

```
periods_unten,1 = Select[Flatten[Differences /@
     (Part[#, All, 1] & /@ {max_unten,sym,1, min_unten,sym,1, max_unten,sym,2, min_unten,sym,2})],
   # > 1 &] // Quantity[#, "Seconds"] &
```

```
{1.634 s, 1.655 s, 1.627 s, 1.611 s, 1.612 s, 1.643 s, 1.62 s, 1.651 s, 1.594 s,
 1.62 s, 1.666 s, 1.606 s, 1.611 s, 1.618 s, 1.631 s, 1.631 s, 1.65 s, 1.61 s,
 1.603 s, 1.639 s, 1.637 s, 1.647 s, 1.623 s, 1.602 s, 1.611 s, 1.64 s, 1.637 s,
 1.632 s, 1.641 s, 1.575 s, 1.623 s, 1.645 s, 1.62 s, 1.613 s, 1.652 s, 1.644 s,
 1.608 s, 1.611 s, 1.641 s, 1.628 s, 1.619 s, 1.653 s, 1.571 s, 1.631 s, 1.641 s,
 1.646 s, 1.622 s, 1.618 s, 1.623 s, 1.634 s, 1.625 s, 1.619 s, 1.629 s, 1.629 s,
 1.641 s, 1.636 s, 1.604 s, 1.652 s, 1.567 s, 1.627 s, 1.668 s, 1.647 s, 1.597 s,
 1.583 s, 1.66 s, 1.615 s, 1.603 s, 1.624 s, 1.658 s, 1.611 s, 1.639 s, 1.616 s,
 1.62 s, 1.62 s, 1.625 s, 1.653 s, 1.633 s, 1.573 s, 1.644 s, 1.618 s, 1.631 s,
 1.631 s, 1.61 s, 1.623 s, 1.63 s, 1.639 s, 1.637 s, 1.647 s, 1.577 s, 1.648 s,
 1.636 s, 1.615 s, 1.637 s, 1.598 s, 1.637 s, 1.613 s, 1.649 s, 1.619 s,
 1.62 s, 1.579 s, 1.652 s, 1.612 s, 1.64 s, 1.638 s, 1.614 s, 1.628 s, 1.619 s,
 1.653 s, 1.604 s, 1.636 s, 1.603 s, 1.613 s, 1.655 s, 1.618 s, 1.623 s,
 1.634 s, 1.625 s, 1.619 s, 1.629 s, 1.629 s, 1.641 s, 1.601 s, 1.608 s,
 1.683 s, 1.595 s, 1.631 s, 1.61 s, 1.639 s, 1.631 s, 1.623 s, 1.62 s, 1.615 s}
```

```
T_unten,1 = Mean[periods_unten,1]; ΔT_unten,1 = StandardDeviation[periods_unten,1];
```

$$\omega_{unten,1} = \frac{2\pi}{T_{unten,1}} \text{ // N}$$

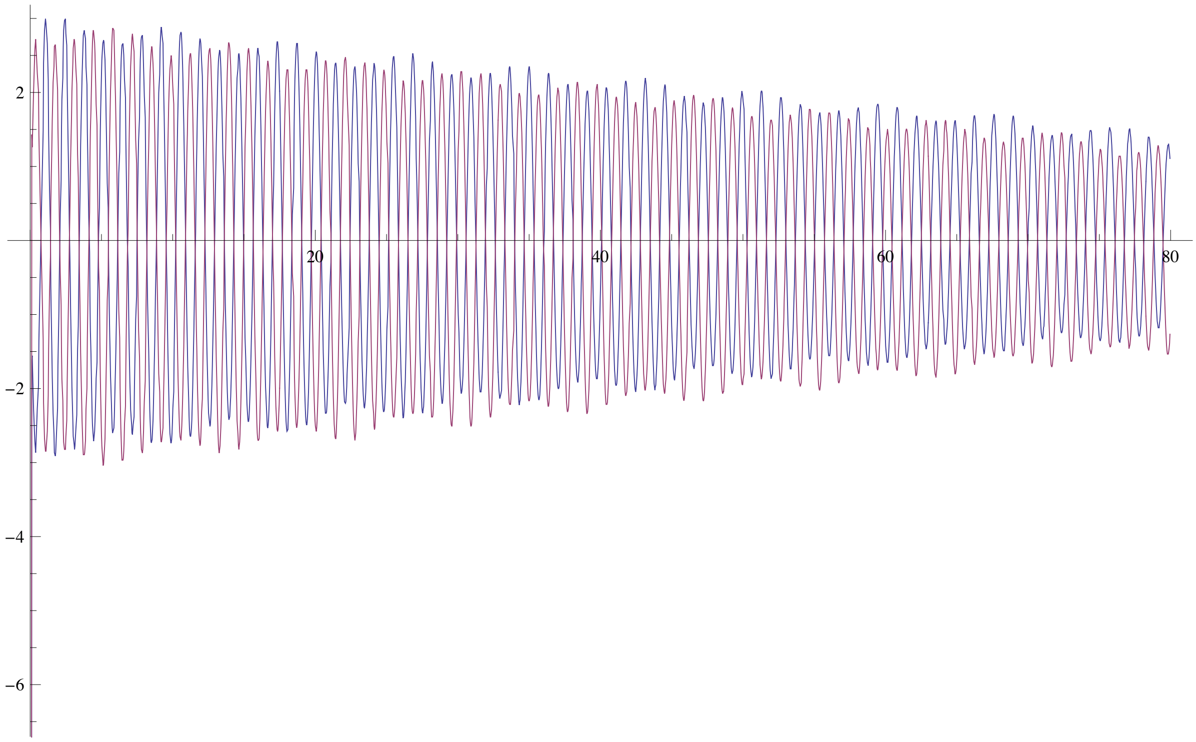
3.86526 per second

$$\Delta\omega_{unten,1} = \frac{2\pi\,\Delta T_{unten,1}}{T_{unten,1}^2} \text{ // N}$$

0.0485766 per second

Antisymmetrische Schwingung

```
ListPlot[{gekoppelt_{unten,asym,1}, gekoppelt_{unten,asym,2}},
 Joined → True, ImageSize → Full]
```



Wir nutzen die ersten 75 Sekunden.

```
gekoppelt_{unten,asym,1} = Select[gekoppelt_{unten,asym,1}, #[[1]] ≤ 75 &];

gekoppelt_{unten,asym,2} = Select[gekoppelt_{unten,asym,2}, #[[1]] ≤ 75 &];
```

Nun bestimmen wir die Frequenz $\omega_{\text{unten},2}$ anhand der Daten von beiden Pendeln.

```
max_{unten,asym,1} = Select[Map[#[[2]] &,
   Select[Partition[gekoppelt_{unten,asym,1}, 3, 1], maxQ[#] &]], #[[2]] > 0 &]
```

```
{{1.074, 2.995}, {2.475, 2.995}, {3.796, 2.839}, {5.151, 2.706}, {6.509, 2.662},
 {7.872, 2.773}, {9.196, 2.884}, {10.578, 2.817}, {11.912, 2.728}, {13.289, 2.573},
 {14.64, 2.528}, {15.958, 2.595}, {17.323, 2.684}, {18.693, 2.662}, {20.07, 2.551},
 {21.415, 2.395}, {22.781, 2.351}, {24.12, 2.395}, {25.485, 2.484},
 {26.836, 2.528}, {28.224, 2.417}, {29.562, 2.24}, {30.908, 2.195},
 {32.276, 2.262}, {33.635, 2.351}, {35.011, 2.351}, {36.366, 2.262},
 {37.708, 2.107}, {39.072, 2.018}, {40.4, 2.062}, {41.765, 2.151},
 {43.152, 2.195}, {44.529, 2.107}, {45.886, 1.951}, {47.214, 1.862},
 {48.551, 1.929}, {49.921, 2.018}, {51.288, 2.018}, {52.633, 1.929},
 {54.019, 1.84}, {55.391, 1.729}, {56.713, 1.751}, {58.093, 1.796},
 {59.447, 1.84}, {60.799, 1.796}, {62.19, 1.685}, {63.542, 1.618}, {64.872, 1.618},
 {66.225, 1.685}, {67.612, 1.707}, {68.962, 1.685}, {70.326, 1.552},
 {71.676, 1.418}, {73.005, 1.418}, {73.066, 1.441}, {74.358, 1.485}}
```

```
min_unten,asym,1 = Select[Map[#[[2]] &,
   Select[Partition[gekoppelt_unten,asym,1, 3, 1], minQ[#] &]], #[[2]] < 0 &]
```

```
{{0.386, -2.866}, {1.765, -2.911}, {3.113, -2.822},
 {4.452, -2.711}, {5.782, -2.6}, {7.172, -2.622}, {8.5, -2.733},
 {9.895, -2.733}, {11.258, -2.644}, {12.612, -2.511}, {13.941, -2.422},
 {15.323, -2.444}, {16.667, -2.533}, {18.012, -2.578}, {19.409, -2.489},
 {20.766, -2.333}, {22.116, -2.2}, {23.453, -2.267}, {24.813, -2.311},
 {26.165, -2.4}, {27.538, -2.333}, {28.918, -2.2}, {30.238, -2.067},
 {31.624, -2.045}, {32.94, -2.134}, {34.31, -2.222}, {35.687, -2.156},
 {37.071, -2.}, {38.427, -1.912}, {39.782, -1.867}, {41.117, -1.956},
 {42.477, -2.045}, {43.824, -2.023}, {45.205, -1.889}, {46.571, -1.734},
 {47.921, -1.69}, {49.238, -1.801}, {50.607, -1.845}, {51.975, -1.867},
 {53.375, -1.734}, {54.713, -1.601}, {56.085, -1.556}, {57.428, -1.623},
 {58.785, -1.69}, {60.17, -1.645}, {61.515, -1.579}, {62.85, -1.468},
 {64.211, -1.401}, {65.53, -1.468}, {66.912, -1.534}, {68.334, -1.49},
 {69.655, -1.423}, {71.022, -1.334}, {72.359, -1.246}, {73.701, -1.312}}
```

```
max_unten,asym,2 = Select[Map[#[[2]] &,
   Select[Partition[gekoppelt_unten,asym,2, 3, 1], maxQ[#] &]], #[[2]] > 0 &]
```

```
{{0.112, 1.431}, {0.386, 2.719}, {1.765, 2.646}, {3.086, 2.719}, {4.427, 2.84},
 {5.782, 2.865}, {7.172, 2.792}, {8.527, 2.622}, {9.895, 2.5}, {11.231, 2.524},
 {12.612, 2.597}, {13.916, 2.67}, {15.323, 2.597}, {16.667, 2.427},
 {18.012, 2.306}, {19.35, 2.306}, {20.71, 2.427}, {22.116, 2.476},
 {23.478, 2.403}, {24.813, 2.306}, {26.165, 2.16}, {27.507, 2.16}, {28.89, 2.257},
 {30.212, 2.281}, {31.624, 2.257}, {32.973, 2.111}, {34.31, 1.99},
 {35.659, 1.965}, {37.011, 2.063}, {38.371, 2.136}, {39.782, 2.111},
 {41.117, 1.941}, {42.477, 1.868}, {43.824, 1.795}, {45.173, 1.893},
 {46.541, 1.965}, {47.893, 1.917}, {49.264, 1.795}, {50.607, 1.674},
 {51.947, 1.625}, {53.316, 1.698}, {54.655, 1.771}, {56.016, 1.722},
 {57.394, 1.65}, {58.75, 1.528}, {60.141, 1.504}, {61.459, 1.504}, {62.85, 1.625},
 {64.211, 1.625}, {65.557, 1.504}, {66.912, 1.382}, {68.275, 1.334},
 {69.592, 1.382}, {70.986, 1.455}, {72.333, 1.455}, {73.701, 1.334}}
```

```
min_unten,asym,2 = Select[Map[#[[2]] &,
   Select[Partition[gekoppelt_unten,asym,2, 3, 1], minQ[#] &]], #[[2]] < 0 &]
```

```
{{1.11, -2.846}, {2.475, -2.822}, {3.796, -2.894}, {5.122, -3.04},
 {6.509, -2.967}, {7.872, -2.87}, {9.196, -2.724}, {10.578, -2.7},
 {11.912, -2.773}, {13.261, -2.87}, {14.64, -2.822}, {16.014, -2.7},
 {17.357, -2.579}, {18.693, -2.53}, {20.07, -2.579}, {21.445, -2.676},
 {22.781, -2.7}, {24.153, -2.554}, {25.517, -2.384}, {26.861, -2.336},
 {28.224, -2.384}, {29.592, -2.506}, {30.939, -2.506}, {32.303, -2.384},
 {33.693, -2.214}, {35.011, -2.165}, {36.366, -2.238}, {37.708, -2.311},
 {39.072, -2.336}, {40.458, -2.214}, {41.793, -2.093}, {43.152, -2.02},
 {44.504, -2.068}, {45.853, -2.165}, {47.24, -2.165}, {48.579, -2.068},
 {48.641, -2.044}, {49.982, -1.947}, {51.288, -1.874}, {52.664, -1.898},
 {54.019, -1.971}, {55.391, -2.02}, {56.739, -1.922}, {58.125, -1.801},
 {59.447, -1.752}, {60.829, -1.752}, {62.157, -1.825}, {63.542, -1.85},
 {64.903, -1.801}, {66.252, -1.679}, {67.612, -1.582}, {68.962, -1.558},
 {70.296, -1.655}, {71.676, -1.704}, {73.066, -1.631}, {74.417, -1.534}}
```

```
periods_{unten,2} =
 Select[Flatten[Differences /@ (Part[#, All, 1] & /@ {max_{unten,asym,1}, min_{unten,asym,1},
      max_{unten,asym,2}, min_{unten,asym,2}})], # > 1 &] // Quantity[#, "Seconds"] &
```

{1.401 s, 1.321 s, 1.355 s, 1.358 s, 1.363 s, 1.324 s, 1.382 s, 1.334 s, 1.377 s,
1.351 s, 1.318 s, 1.365 s, 1.37 s, 1.377 s, 1.345 s, 1.366 s, 1.339 s, 1.365 s,
1.351 s, 1.388 s, 1.338 s, 1.346 s, 1.368 s, 1.359 s, 1.376 s, 1.355 s, 1.342 s,
1.364 s, 1.328 s, 1.365 s, 1.387 s, 1.377 s, 1.357 s, 1.328 s, 1.337 s, 1.37 s,
1.367 s, 1.345 s, 1.386 s, 1.372 s, 1.322 s, 1.38 s, 1.354 s, 1.352 s, 1.391 s,
1.352 s, 1.33 s, 1.353 s, 1.387 s, 1.35 s, 1.364 s, 1.35 s, 1.329 s, 1.292 s,
1.379 s, 1.348 s, 1.339 s, 1.33 s, 1.39 s, 1.328 s, 1.395 s, 1.363 s, 1.354 s,
1.329 s, 1.382 s, 1.344 s, 1.345 s, 1.397 s, 1.357 s, 1.35 s, 1.337 s, 1.36 s,
1.352 s, 1.373 s, 1.38 s, 1.32 s, 1.386 s, 1.316 s, 1.37 s, 1.377 s, 1.384 s,
1.356 s, 1.355 s, 1.335 s, 1.36 s, 1.347 s, 1.381 s, 1.366 s, 1.35 s, 1.317 s,
1.369 s, 1.368 s, 1.4 s, 1.338 s, 1.372 s, 1.343 s, 1.357 s, 1.385 s, 1.345 s,
1.335 s, 1.361 s, 1.319 s, 1.382 s, 1.422 s, 1.321 s, 1.367 s, 1.337 s, 1.342 s,
1.379 s, 1.321 s, 1.341 s, 1.355 s, 1.39 s, 1.355 s, 1.368 s, 1.336 s, 1.381 s,
1.304 s, 1.407 s, 1.344 s, 1.345 s, 1.338 s, 1.36 s, 1.406 s, 1.362 s, 1.335 s,
1.352 s, 1.342 s, 1.383 s, 1.322 s, 1.412 s, 1.349 s, 1.337 s, 1.349 s, 1.352 s,
1.36 s, 1.411 s, 1.335 s, 1.36 s, 1.347 s, 1.349 s, 1.368 s, 1.352 s, 1.371 s,
1.343 s, 1.34 s, 1.369 s, 1.339 s, 1.361 s, 1.378 s, 1.356 s, 1.391 s, 1.318 s,
1.391 s, 1.361 s, 1.346 s, 1.355 s, 1.363 s, 1.317 s, 1.394 s, 1.347 s, 1.368 s,
1.365 s, 1.321 s, 1.326 s, 1.387 s, 1.363 s, 1.324 s, 1.382 s, 1.334 s, 1.349 s,
1.379 s, 1.374 s, 1.343 s, 1.336 s, 1.377 s, 1.375 s, 1.336 s, 1.372 s, 1.364 s,
1.344 s, 1.363 s, 1.368 s, 1.347 s, 1.364 s, 1.39 s, 1.318 s, 1.355 s, 1.342 s,
1.364 s, 1.386 s, 1.335 s, 1.359 s, 1.352 s, 1.349 s, 1.387 s, 1.339 s, 1.341 s,
1.306 s, 1.376 s, 1.355 s, 1.372 s, 1.348 s, 1.386 s, 1.322 s, 1.382 s, 1.328 s,
1.385 s, 1.361 s, 1.349 s, 1.36 s, 1.35 s, 1.334 s, 1.38 s, 1.39 s, 1.351 s}

$T_{unten,2}$ = Mean[$periods_{unten,2}$]; $\Delta T_{unten,2}$ = StandardDeviation[$periods_{unten,2}$];

$$\omega_{unten,2} = \frac{2\pi}{T_{unten,2}} \text{ // N}$$

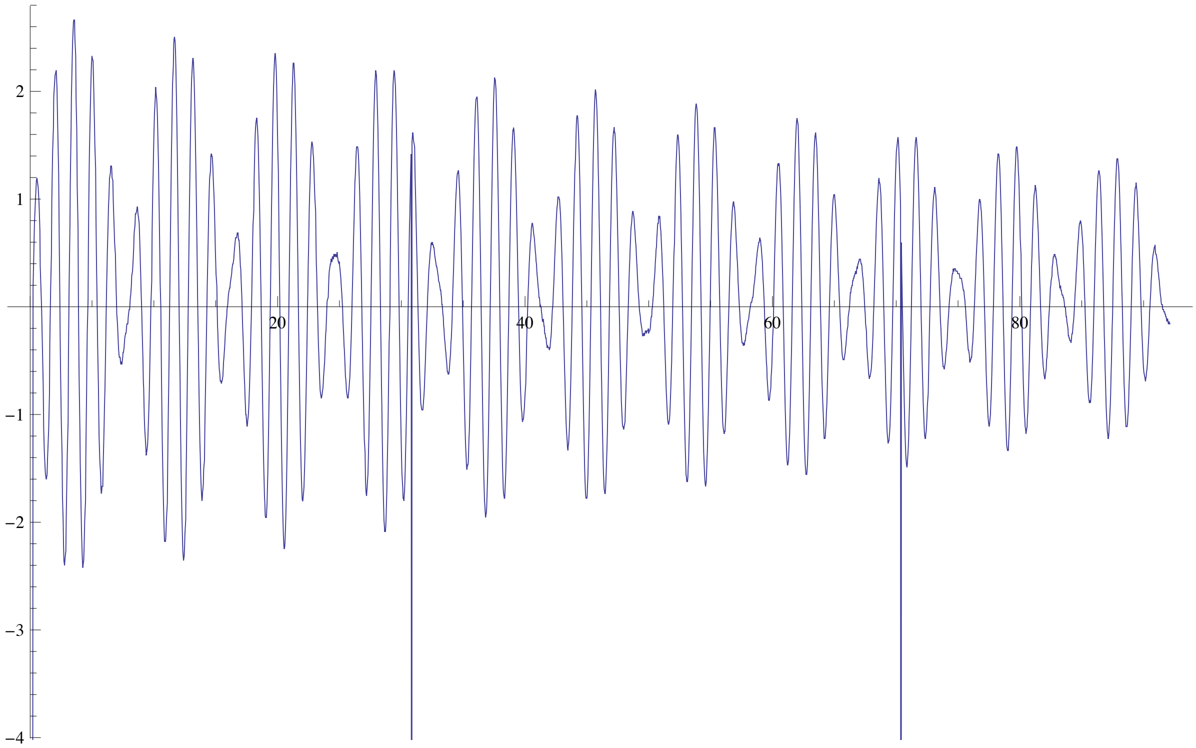
4.63042 per second

$$\Delta\omega_{unten,2} = \frac{2\pi\,\Delta T_{unten,2}}{T_{unten,2}^{2}} \text{ // N}$$

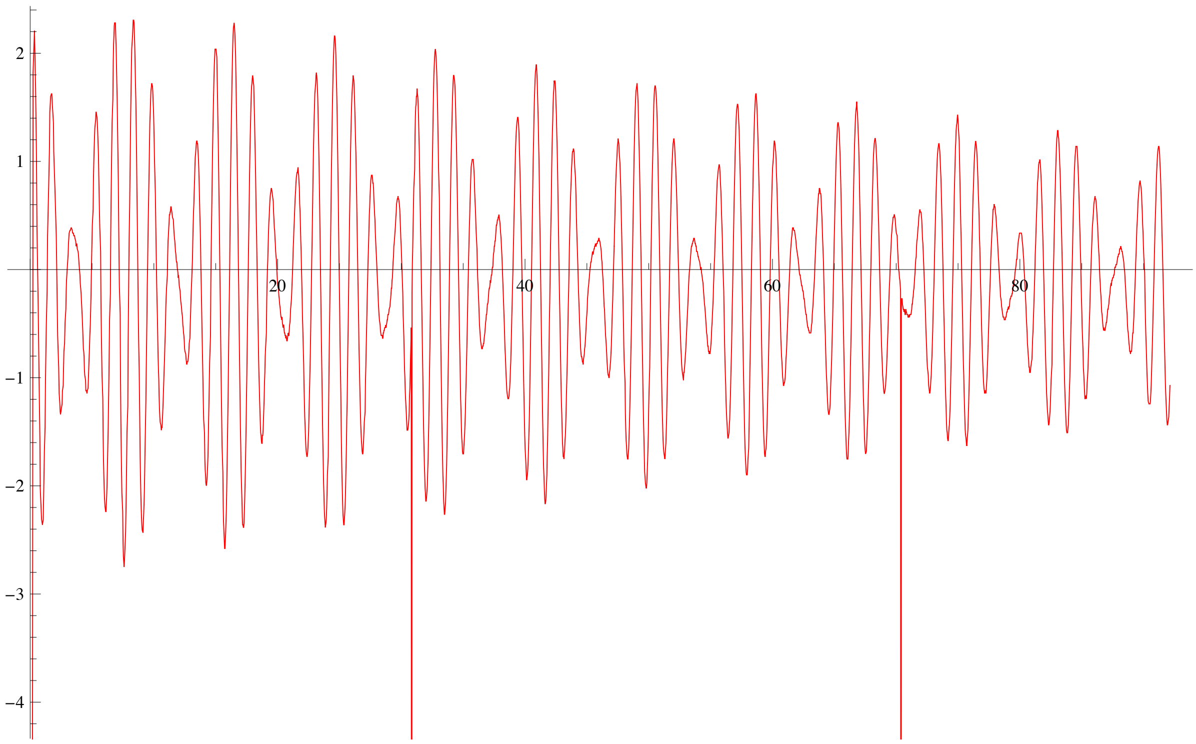
0.0778968 per second

Schwebung

```
ListPlot[gekoppelt_{unten,schweb,1}, Joined → True, ImageSize → Full]
```




```
ListPlot[gekoppelt_{unten,schweb,2}, Joined → True, ImageSize → Full, PlotStyle → Red]
```



Zunächst ermitteln wir $\omega_{\text{unten},II}$:

```
peaks_unten,schweb,1 = {{3.522, 2.642}, {11.92, 2.548}, {20.52, 2.396},
  {28.61, 2.266}, {37.62, 2.125}, {45.71, 2.007}, {53.9, 1.878},
  {61.88, 1.725}, {70.89, 1.596}, {78.98, 1.502}, {87.99, 1.361}}
```

```
{{3.522, 2.642}, {11.92, 2.548}, {20.52, 2.396},
 {28.61, 2.266}, {37.62, 2.125}, {45.71, 2.007}, {53.9, 1.878},
 {61.88, 1.725}, {70.89, 1.596}, {78.98, 1.502}, {87.99, 1.361}}
```

```
peaks_unten,schweb,2 =
 {{7.925, -2.756}, {16.42, 2.268}, {24.61, 2.151}, {32.8, 2.022}, {41.92, -2.182},
  {50.01, -2.03}, {58.61, -1.925}, {66.9, 1.565}, {74.99, 1.413}, {83.18, 1.26}}
```

```
{{7.925, -2.756}, {16.42, 2.268}, {24.61, 2.151}, {32.8, 2.022}, {41.92, -2.182},
 {50.01, -2.03}, {58.61, -1.925}, {66.9, 1.565}, {74.99, 1.413}, {83.18, 1.26}}
```

```
periods_unten,II = Select[Flatten[
    Differences /@ (Part[#, All, 1] & /@ {peaks_unten,schweb,1, peaks_unten,schweb,2})],
   # > 1 &] // Quantity[#, "Seconds"] &
```

```
{8.398 s, 8.6 s, 8.09 s, 9.01 s, 8.09 s, 8.19 s, 7.98 s, 9.01 s, 8.09 s,
 9.01 s, 8.495 s, 8.19 s, 8.19 s, 9.12 s, 8.09 s, 8.6 s, 8.29 s, 8.09 s, 8.19 s}
```

```
T_unten,II = Mean[periods_unten,II]; ΔT_unten,II = StandardDeviation[periods_unten,II];
```

$$\omega_{\text{unten,II}} = \frac{\pi}{T_{\text{unten,II}}} \; // \; N$$

0.373711 per second

$$\Delta\omega_{\text{unten,II}} = \frac{\pi \, \Delta T_{\text{unten,II}}}{T_{\text{unten,II}}^2} \; // \; N$$

0.0167924 per second

Nun bestimmen wir $\omega_{\text{unten,I}}$. Dazu nutzen wir die ersten zwei Blöcke des ersten Pendels (0-5 s und 10-15 s) und den ersten Block des zweiten Pendels (5-10 s):

```
max_unten,schweb,1,1 = Select[
  Map[#[[2]] &, Select[Partition[Select[gekoppelt_unten,schweb,1, #[[1]] ≤ 5 &], 3, 1],
    maxQ[#] &]], #[[2]] > 0 &]
```

```
{{0.55, 1.196}, {2.109, 2.195}, {3.524, 2.662}}
```

```
min_unten,schweb,1,1 = Select[
  Map[#[[2]] &, Select[Partition[Select[gekoppelt_unten,schweb,1, #[[1]] ≤ 5 &], 3, 1],
    minQ[#] &]], #[[2]] < 0 &]
```

```
{{1.304, -1.601}, {2.781, -2.4}, {4.25, -2.422}}
```

```
max_unten,schweb,1,2 =
 Select[Map[#[[2]] &, Select[Partition[Select[gekoppelt_unten,schweb,1,
      10 ≤ #[[1]] ≤ 15 &], 3, 1], maxQ[#] &]], #[[2]] > 0 &]
```

```
{{10.145, 2.04}, {11.663, 2.506}, {13.145, 2.306}, {14.631, 1.418}}
```

```
min_unten,schweb,1,2 =
 Select[Map[#[[2]] &, Select[Partition[Select[gekoppelt_unten,schweb,1,
      10 ≤ #[[1]] ≤ 15 &], 3, 1], minQ[#] &]], #[[2]] < 0 &]
```

```
{{10.941, -2.178}, {12.397, -2.356}, {13.886, -1.801}}
```

```
max_{unten,schweb,2} = Select[Map[#[[2]] &,
    Select[Partition[Select[gekoppelt_{unten,schweb,2}, 5 ≤ #[[1]] && #[[1]] ≤ 10 &], 3, 1],
     maxQ[#] &]], #[[2]] > 0 &]
```

{{5.319, 1.455}, {6.823, 2.281}, {8.328, 2.306}, {9.828, 1.722}}

```
min_{unten,schweb,2} = Select[Map[#[[2]] &,
    Select[Partition[Select[gekoppelt_{unten,schweb,2}, 5 ≤ #[[1]] && #[[1]] ≤ 10 &], 3, 1],
     minQ[#] &]], #[[2]] < 0 &]
```

{{6.13, -2.238}, {7.591, -2.749}, {9.121, -2.433}}

```
periods_{unten,I} = Select[
    Flatten[Differences /@ (Part[#, All, 1] & /@ {max_{unten,schweb,1,1}, min_{unten,schweb,1,1},
          max_{unten,schweb,1,2}, min_{unten,schweb,1,2}, max_{unten,schweb,2}, min_{unten,schweb,2}})],
    # > 1 &] // Quantity[#, "Seconds"] &
```

{1.559 s, 1.415 s, 1.477 s, 1.469 s, 1.518 s, 1.482 s,
 1.486 s, 1.456 s, 1.489 s, 1.504 s, 1.505 s, 1.5 s, 1.461 s, 1.53 s}

```
T_{unten,I} = Mean[periods_{unten,I}]; ΔT_{unten,I} = StandardDeviation[periods_{unten,I}];
```

$$\omega_{unten,I} = \frac{2\pi}{T_{unten,I}} \;//\; N$$

4.21872 per second

$$\Delta\omega_{unten,I} = \frac{2\pi\,\Delta T_{unten,I}}{T_{unten,I}^2} \;//\; N$$

0.0992903 per second

Vergleich der Frequenzen

$$\omega_{unten,I,theor} = \frac{1}{2}(\omega_{unten,1} + \omega_{unten,2})$$

4.24784 per second

$$\Delta\omega_{unten,I,theor} = \frac{1}{2}(\Delta\omega_{unten,1} + \Delta\omega_{unten,2})$$

0.0632367 per second

$$\omega_{unten,II,theor} = \frac{1}{2}(\omega_{unten,2} - \omega_{unten,1})$$

0.382582 per second

$$\Delta\omega_{unten,II,theor} = \frac{1}{2}(\Delta\omega_{unten,2} + \Delta\omega_{unten,1})$$

0.0632367 per second

```
Grid[{{Null, Text["Theoretischer Wert"], Text["Gemessener Wert"]},
  {Text["ω_{unten,I}"], NumberForm[ω_{unten,I,theor}, 3] ± NumberForm[Δω_{unten,I,theor}, 1],
   NumberForm[ω_{unten,I}, 2] ± NumberForm[Δω_{unten,I}, 1]},
  {Text["ω_{unten,II}"], NumberForm[ω_{unten,II,theor}, 2] ± NumberForm[Δω_{unten,II,theor}, 1],
   NumberForm[ω_{unten,II}, 2] ± NumberForm[Δω_{unten,II}, 1]}}, Frame → All]
```

	Theoretischer Wert	Gemessener Wert
$\omega_{\text{unten,I}}$	4.25 per second ± 0.06 per second	4.2 per second ± 0.1 per second
$\omega_{\text{unten,II}}$	0.38 per second ± 0.06 per second	0.37 per second ± 0.02 per second

Kopplungsgrad

$$\kappa_{\text{unten}} = \frac{\omega_{\text{unten},2}^2 - \omega_{\text{unten},1}^2}{\omega_{\text{unten},2}^2 + \omega_{\text{unten},1}^2}$$

0.178681

$$\Delta\kappa_{\text{unten}} = \frac{4\,\omega_{\text{unten},1}\,\omega_{\text{unten},2}}{\left(\omega_{\text{unten},2}^2 + \omega_{\text{unten},1}^2\right)^2} \sqrt{(\omega_{\text{unten},1}\,\Delta\omega_{\text{unten},2})^2 + (\omega_{\text{unten},2}\,\Delta\omega_{\text{unten},1})^2}$$

0.0203284

Verhältnisse der Kopplungsgrade

Die angegebene Näherung für die Kopplungsquadrate impliziert, daß sich die Verhältnisse der Kopplungsgrade wie die Verhältnisse der Längenquadrate verhalten.

Oben–Mitte

$$V_{oben,mitte} = \frac{\kappa_{oben}}{\kappa_{mitte}}$$

0.370362

$$\Delta V_{oben,mitte} = \sqrt{\left(\frac{\Delta\kappa_{oben}}{\kappa_{mitte}}\right)^2 + \left(\frac{\kappa_{oben}\,\Delta\kappa_{mitte}}{\kappa_{mitte}^2}\right)^2}$$

0.371067

$$V_{oben,mitte,approx} = \frac{l_{oben}^2}{l_{mitte}^2}$$

0.125911

$$\Delta V_{oben,mitte,approx} = \sqrt{\left(\frac{2\,l_{oben}\,\Delta l_{oben}}{l_{mitte}^2}\right)^2 + \left(\frac{2\,l_{oben}^2\,\Delta l_{mitte}}{l_{mitte}^3}\right)^2}$$

0.00485827

```
Grid[
 {{Text["Verhältnis"], Text["aus Frequenzmessung"], Text["aus Längenmessung"]},
  {Text["κoben/κmitte"], NumberForm[Voben,mitte, 1] ± NumberForm[ΔVoben,mitte, 1],
   NumberForm[Voben,mitte,approx, 3] ± NumberForm[ΔVoben,mitte,approx, 1]}}, Frame → All]
```

Verhältnis	aus Frequenzmessung	aus Längenmessung
$\frac{\kappa_{oben}}{\kappa_{mitte}}$	0.4 ± 0.4	0.126 ± 0.005

Hier läßt sich tatsächlich nicht viel sagen, da der Fehler auf das Verhältnis dank großer Frequenzmeßfehler so groß ist wie das Verhältnis selbst.

Oben–Unten

$$V_{oben,unten} = \frac{\kappa_{oben}}{\kappa_{unten}}$$

0.157309

$$\Delta V_{oben,unten} = \sqrt{\left(\frac{\Delta\kappa_{oben}}{\kappa_{unten}}\right)^2 + \left(\frac{\kappa_{oben}\,\Delta\kappa_{unten}}{\kappa_{unten}^2}\right)^2}$$

0.145207

$$V_{oben,unten,approx} = \frac{l_{oben}^2}{l_{unten}^2}$$

0.0325181

$$\Delta V_{oben,unten,approx} = \sqrt{\left(\frac{2\, l_{oben}\, \Delta l_{oben}}{l_{unten}^2}\right)^2 + \left(\frac{2\, l_{oben}^2\, \Delta l_{unten}}{l_{unten}^3}\right)^2}$$

0.00120155

```
Grid[
 {{Text["Verhältnis"], Text["aus Frequenzmessung"], Text["aus Längenmessung"]},
  {Text["κoben/κunten"], NumberForm[Voben,unten, 1] ± NumberForm[ΔVoben,unten, 1],
   NumberForm[Voben,unten,approx, 2] ± NumberForm[ΔVoben,unten,approx, 1]}}, Frame → All]
```

Verhältnis	aus Frequenzmessung	aus Längenmessung
$\frac{\kappa_{oben}}{\kappa_{unten}}$	0.2 ± 0.1	0.033 ± 0.001

Auch hier haben wir einen sehr großen Fehler bei dem Wert, der sich aus der Frequenzmessung ergibt, und können daher kaum etwas über die Qualität der Approximation aussagen.

Mitte–Unten

$$V_{mitte,unten} = \frac{\kappa_{mitte}}{\kappa_{unten}}$$

0.424743

$$\Delta V_{mitte,unten} = \sqrt{\left(\frac{\Delta\kappa_{mitte}}{\kappa_{unten}}\right)^2 + \left(\frac{\kappa_{mitte}\, \Delta\kappa_{unten}}{\kappa_{unten}^2}\right)^2}$$

0.179014

$$V_{mitte,unten,approx} = \frac{l_{mitte}^2}{l_{unten}^2}$$

0.258264

$$\Delta V_{mitte,unten,approx} = \sqrt{\left(\frac{2\, l_{mitte}\, \Delta l_{mitte}}{l_{unten}^2}\right)^2 + \left(\frac{2\, l_{mitte}^2\, \Delta l_{unten}}{l_{unten}^3}\right)^2}$$

0.00373807

```
Grid[
 {{Text["Verhältnis"], Text["aus Frequenzmessung"], Text["aus Längenmessung"]},
  {Text["κmitte/κunten"], NumberForm[Vmitte,unten, 1] ± NumberForm[ΔVmitte,unten, 1],
   NumberForm[Vmitte,unten,approx, 3] ± NumberForm[ΔVmitte,unten,approx, 1]}}, Frame → All]
```

Verhältnis	aus Frequenzmessung	aus Längenmessung
$\frac{\kappa_{mitte}}{\kappa_{unten}}$	0.4 ± 0.2	0.258 ± 0.004

Hier haben wir Werte, die zumindest im 1 σ-Bereich zur Deckung kommen. Sehr aufschlußreich ist diese Aussage indes auch nicht.