P.1

$y(x) = y_0 + \int_{x_0}^{x} f(x')\,dx'$ ... allg. Lsg durch Integration

$$\Rightarrow y(x) = 1 + \int_1^x (x'^3 + \cos(x'))\,dx'$$

$$= 1 + \left[\frac{x'^4}{4} + \sin(x')\right]_1^x$$

$$= 1 + \left(\frac{x^4}{4} + \sin(x)\right) - \left(\frac{1}{4} + \sin(1)\right)$$

$$= \frac{3}{4} - \sin(1) + \frac{x^4}{4} + \sin(x)$$

P2

$y'(x) = f(x)\,y(x)$

$$\Rightarrow \frac{dy}{dx} = f(x)\,y(x)$$

$$\Rightarrow \int_{y(x_0)}^{y(x)} \frac{dy}{y} = \int_{x_0}^{x} f(x')\,dx'$$

$$\Rightarrow \ln(y(x)) - \ln(y_0) = \int_{x_0}^{x} f(x')\,dx'$$

$$\Rightarrow y(x) = y_0 \exp\left[\int_{x_0}^{x} f(x')\,dx'\right]$$

z.B. $f(x) = 3x^2$ $\Rightarrow$ $y(x) = y_0 \exp\left[\int_{x_0}^{x} 3x'^2\,dx'\right]$

$$\overset{y_0=1}{\underset{x_0=0}{=}} \exp[x^3]$$